V

Making Financial Decisions

In this section we look at ways to quantitatively analyze financial decisions. This section includes the following chapters:

17

Tools for Making Financial Decisions

In this chapter you learn ten quantitative methods that may be used to analyze financial alternatives and to choose the alternative that is best for the company. Without some quantitative method it is often hard for managers to determine which option is best. An understanding of these skills is a necessity for any manager who has to decide where to invest limited capital.

Financial decisions can be viewed as the selection of one or more alternatives from a pool of alternatives. The pool of alternatives may include independent alternatives, mutually exclusive alternatives, contingent alternatives, and the "do nothing" alternative.

Independent alternatives are alternatives where the acceptance of one alternative does not, in and of itself, preclude the selection of the other alternatives. For example, investing part of your surplus capital in heavy equipment does not preclude investing the remaining portion of your surplus capital in a real estate development. Provided there is sufficient capital, you can invest in both of these alternatives.

Mutually exclusive alternatives are alternatives where the acceptance of one of the alternatives precludes investing in the other alternatives. For example, if your company needs to purchase a single backhoe and has narrowed the selection down to three backhoes, when your company purchases one of the backhoes, the need for a backhoe has been fulfilled and the purchase of a second backhoe is no longer an option. Once you have selected one of the three alternatives, the other two alternatives are no longer viable.

Contingent alternatives are alternatives that may be selected only after another alternative has been selected. For example, if your company is looking at purchasing a dump truck and as a second alternative purchasing a dumping trailer to be pulled behind the dump truck, the purchase of the dumping trailer is a contingent alternative because for the dumping trailer to be useful, you must first purchase the dump truck.

The "do nothing" alternative is simply another way of saying that all of the alternatives have been rejected.

Decisions are limited not only by the types of alternatives in the pool but also by external constraints. A common external constraint is the limited supply of capital. With a limited supply of capital, there may be a dozen independent alternatives, but a company may invest in only a fraction of the alternatives because there is not enough capital to invest in all of the alternatives. Another common external constraint is contractual obligations. If a company has a contract to perform excavation work that requires the use of a special backhoe attachment—which the company does not own—management must make a decision that makes the attachment available for use on the contract. When making decisions, both the external constraints and available alternatives must be taken into account.

To use the quantitative methods described in this chapter, one must reduce all decisions to a set of mutually exclusive alternatives. When deciding whether to invest in a single alternative, the decision has two mutually exclusive alternatives: investment in the alternative and the "do nothing" alternative; in other words, reject the alternative. In this situation, the manager must decide if the alternative meets the minimum requirement for profitability. When deciding between mutually exclusive alternatives, the manager must decide which of the alternatives is most financially advantageous while meeting the external constraints. If investment in none of the alternatives is an option, the "do nothing" alternative should be included in the pool of alternatives.

When faced with a decision, which includes contingent or independent alternatives, the decision must be reduced to a set of mutually exclusive alternatives before the alternative may be analyzed quantitatively. This is done by listing all of the possible combinations of alternatives and eliminating infeasible alternatives. A combination of alternatives is infeasible when they (1) contain two or more mutually exclusive alternatives, (2) contain a contingent alternative without containing the alternative that the contingent alternative is contingent on, or (3) do not meet the requirements of the external factors. Converting the decision to a set of mutually exclusive alternatives containing all feasible combinations of alternatives reduces the decision-making process to the selection of a single, best combination of alternatives from a group of all feasible combinations of alternatives. The conversion of alternatives to mutually exclusive alternatives is shown in Example 17-1.

Example 17-1: A manager has up to $200,000 available to invest in new construction equipment for a construction company. The potential new construction equipment has been reduced to the alternatives shown in Table 17-1. The manager must purchase a backhoe to complete an existing contract. A second backhoe is not needed. Prepare a list of the mutually exclusive alternatives and identify which alternatives are not feasible.

Solution: Here the manager is faced with two external constraints: The capital available for investment is $200,000 and only one backhoe needs to be purchased. For the manager to analyze the alternatives quantitative, the

TABLE 17-1 Alternatives

No.	Description	Cost ($)
1	Backhoe 1	100,000
2	Backhoe 2	120,000
3	Dump Truck	65,000
4	Dumping Trailer for the Dump Truck	25,000

alternatives must be converted to a group of mutually exclusive combinations of the alternatives that include only the feasible alternatives. Alternatives 1 and 2 are mutually exclusive in that only one backhoe is needed. Alternatives 1 and 3 are independent as are Alternatives 2 and 3. Alternative 4 is contingent on the purchase of Alternative 3. Combinations of the alternatives that do not include Alternative 1 or 2 are unacceptable because the manager must purchase a backhoe. Combinations of the alternatives that include both Alternatives 1 and 2 are unacceptable because the manager needs to purchase only one backhoe, not two. Combinations of the alternatives that include Alternative 4 but do not include Alternative 3 are unacceptable because Alternative 4 is contingent on the selection of Alternative 3. Combinations of alternatives that cost more than $200,000 are unacceptable because they exceed the capital available for investment. Table 17-2 contains a list of all possible combinations of alternatives and identifies those that are acceptable.

TABLE 17-2 Combination of Alternatives

Alternatives	Acceptable	Reasoning
Do nothing	No	Does not contain 1 or 2
1	Yes	
2	Yes	
3	No	Does not contain 1 or 2
4	No	Does not contain 1 or 2
1 and 2	No	Selects both 1 and 2
1 and 3	Yes	
1 and 4	No	Selects 4 without 3
2 and 3	Yes	
2 and 4	No	Selects 4 without 3
3 and 4	No	Does not contain 1 or 2
1, 2 and 3	No	Selects both 1 and 2
1, 2 and 4	No	Selects both 1 and 2
1, 3 and 4	Yes	
2, 3 and 4	No	Exceeds budget
1, 2, 3 and 4	No	Selects both 1 and 2, exceeds budget

Thus, the alternatives are reduced to five mutually exclusive combinations of alternatives. These five combinations now may be quantitatively analyzed.

When making decisions, the decision maker may make each decision separately or group them together into one more complex decision. The manager in Example 17-1 has three decisions to make: which backhoe to purchase from Alternatives 1 and 2; should the company invest in a dump truck; and, if the company invests in a dump truck, should it invest in a dumping trailer? The strength of making each of these decisions separately is the simplicity of each of the decisions. In each case there are only two alternatives to select between: Do we purchase Backhoe 1 or Backhoe 2, do we invest in a dump truck or do we do nothing, and do we invest in a dumping trailer or do we do nothing? The weakness of making each of these decisions separately is that the manager does not try to invest the limited supply of capital in the most profitable alternatives but simply selects from the available alternatives for each of the decisions. This may result in a manager choosing to invest in an alternative because it meets the minimum requirements for profitability, only to have to pass up a second, more profitable alternative because of the lack of capital, capital that would have been available if the manager had not chosen to invest in the first alternative. For example, if the manager in Example 17-1 decides to purchase Backhoe 2 because it is more profitable than Backhoe 1, it precludes the manager from investing in the dumping trailer. The most profitable combination may be to purchase Backhoe 1, the dump truck, and the dumping trailer; however, this option has been eliminated because of the decision to purchase Backhoe 2.

By combining the decisions as to which backhoe to purchase, whether to purchase a dump truck, and whether to purchase a dumping trailer into one decision, the profit on the available cash may be optimized. The strength of combining decisions into one decision is that interaction between the decisions is taken into account to reach the best combination of decisions. The weakness of combining decisions into one decision is that the complexity of the decision increases. The number of combinations of alternatives is usually greater than the number of alternatives. In Example 17-1, it has increased from four to five. It is not always possible to combine all of the decisions into one complex decision, but where possible combining decisions will lead to more profitable decisions.

When faced with a decision, a decision maker must decide which alternatives, if any, are the most financially advantageous for the company. A number of quantitative methods have been developed to assist the decision maker in comparing the alternatives. All of the qualitative methods discussed in this chapter are based on some measure of equivalence. Because equivalence is a function of the sizes of the cash flows, the timing of the cash flows, and the interest rate, to qualitatively compare alternatives, we must eliminate two of the variables. In this chapter, we look at qualitative methods that compare alternatives using each of the three variables while eliminating the other two. The methods covered in this chapter along with their basis of comparison are shown in Table 17-3.

TABLE 17-3 Qualitative Methods

METHOD	BASIS OF COMPARISON
Net Present Value	Size of Cash Flows
Incremental Net Present Value	Size of Cash Flows
Future Worth	Size of Cash Flows
Annual Equivalent	Size of Cash Flows
Rate of Return	Interest Rate
Incremental Rate of Return	Interest Rate
Capital Recover with Interest	Time
Payback Period without Interest	Time
Payback Period with Interest	Time
Project Balance	Size of Cash Flows and Time

To quantitatively evaluate each of the mutually exclusive combinations of alternatives, cost must be assigned to each alternative. These costs should be identified as accurately as possible. One must keep in mind that the results of the quantitative analysis are only as good as the data used in the analysis. Often the data used in the quantitative analysis are estimates of cash flows occurring years in the future and can easily change based on economic and political environment at the time these cash flows occur.

To use the equations in Chapter 15 to perform quantitative analysis, one must assume that all cash flows incurred during a period occur at the end of the period. When using a period length of one year in the analysis, one ignores the interest on the cash flow for part of a year. The accuracy of the analysis can be increased by shortening the period from years to months; however, this increase in accuracy requires that all cash flows must be tied to the shorter periods. This change in period length can increase the data by twelvefold and greatly increase the complexity of the calculations. In most cases, the increase in error as a result of trying to guess at which month the cash flow will occur combined with the error introduced by estimating future costs on a monthly basis has a much greater impact on the accuracy of the results than is gained analyzing the alternative on a monthly basis rather than an annual basis. Unless future cash flows can be accurately projected as to their size and timing—for example, they are set forth by contract—using a period of one year for the analysis produces the best results.

SUNK COSTS

Costs that have already been spent are known as sunk costs. Sunk costs also include all costs that have been committed to be spent for which the commitment cannot be canceled. When analyzing investment alternatives, sunk costs should not be included because they must be paid regardless of which alternative is chosen. For example, if a company has just spent $10,000 on an engine overhaul for a front-end

loader and is now looking at replacing the loader versus continuing to use the loader, one would not include the cost of the engine overhaul in the alternative that continues to use the loader. This is because the cost of the engine overhaul has been incurred and will still be a cost regardless of which of the two alternatives are chosen. However, if the front-end loader was in need of an engine overhaul and a company was looking at replacing the loader versus continuing to use the loader, one would include the cost of the engine overhaul in the alternative that continues to use the loader. This is because the cost of the engine overhaul has not been incurred and will only be incurred if the company continues to use the loader.

MARR (Minimum Attractive Rate of Return)

When comparing alternatives based on the size of the cash flow or time, the variable of interest rate must be eliminated from the equivalence equation. To do this one must determine at what interest rate equivalence will be established. Additionally, when deciding whether to proceed with an investment based on the interest rate paid on the investment (also known as the return on investment), one must have a minimum acceptable interest rate to act as a cutoff point. The interest rate used in determining equivalence during the decision-making process and the interest rate used as the minimum acceptable interest rate is known as the minimum attractive rate of return or MARR. The MARR is the lowest rate of return or interest paid by an investment that is acceptable to the investor. The MARR is different for each company.

The MARR is based on the cost of capital plus a profit. For a company that has to borrow money to make investments, the cost of capital is the effective annual interest rate at which the money will have to be borrowed. For example, if a company were looking at replacing a backhoe by taking out a loan, the cost of capital would be the effective annual interest rate on the loan. For a company that will use existing funds to make investments, the cost of capital is the effective annual interest rate on financial instruments that it would store the funds in if it did not make the investment. For example, if a company were looking at replacing a backhoe by turning in a certificate of deposit, the cost of capital is the effective annual interest rate paid on the certificate of deposit. In this case, if the company were to replace the backhoe, rather than paying interest on a loan, it would lose the interest it would have received on the certificate of deposit. This loss of interest is known as the opportunity cost. By replacing the backhoe, the company has lost the opportunity to invest in and receive the interest from the certificate of deposit. The MARR should always be above the company's cost of capital. If the MARR is below the cost of capital, the company is paying or losing more in interest than is generated by investments meeting the MARR and the company would be better off financially not making the investment.

Determining the cost of capital is relatively easy when dealing with financial instruments with fixed interest rates. When dealing with financial instruments with variable interest rates, the cost of capital must include any anticipated changes in the interest rate.

The second component of the MARR is profit. Suppose you were given two options to invest your savings. The first option is government-backed Treasury bonds that pay a guaranteed interest rate of 4% and have little risk of loss. The second option is to invest in a real estate development that is expected to pay an interest rate of 4% but has a significant risk of loss of the invested funds. Which would you invest in? Most people would invest in the Treasury bonds because why risk losing your money on the real estate investment when you can earn the same interest rate on the Treasury bonds? To make the second investment attractive, the potential for increased profits or returns are needed to offset the risk. Determining the desired return level is quite subjective and is often set by management. The MARR should be set at a rate at which the company has many opportunities to invest. Setting the MARR too high will result in the company passing up many good investments. Setting the MARR too low will result in the company making too many bad or marginal investments.

Given that the MARR is the sum of the cost of capital and the subjective profit markup, the MARR is calculated using the following equation:

$$\text{MARR} = \text{Cost of Capital} + \text{Profit Margin} \qquad (18\text{-}1)$$

Example 17-2:   Determine the MARR for a company that can borrow funds at 8.5% and requires 7% profit margin or return.

Solution: The MARR is determined by using Eq. (18-1):

$$\text{MARR} = 8.5\% + 7\% = 15.5\%$$

ADJUSTING LIFE SPANS

When comparing alternatives based on the size of the cash flow or interest rate, the variable of time must be eliminated from the equivalence equation. To do this, not only do all cash flows need to be converted to a common point or set of points in time but also the life spans of the alternatives must be equal. Failure to compare the alternatives over equal life spans ignores what happens to the shorter alternative at the end of its life.

Study Period

The time over which alternatives are compared is known as the study period. Determining the study period is as difficult as determining the MARR. The study period is selected in one of the following ways:

1. The study period may be set by company policy. For example, a company may have a policy that all investments are compared over a five-year period.
2. The study period may be matched to the life of one of the investments. A common practice is to match the study period to the alternative with either

the longest or shortest life; however, the study period may be set to the life of one of the other alternatives.

3. The study period may be set to the least common denominator of the alternative lives. Using this method allows one to assume that they continue purchasing each of the alternatives until their lives end at the same point in time. The least common denominator is the smallest number when divided by the useful lives of each of the alternatives results in a whole number. For example, when comparing alternatives that have lives of 3 years, 4 years, and 6 years, the least common dominator is 12 years. A period of less than 6 years does not cover the life of at least one of the alternatives. When divided by a period of 4 years, 6 years does not produce a whole number. Numbers between 7 and 11 years do not produce whole numbers when divided by 6 years. Twelve years produces the whole numbers of 4, 3, and 2 when divided by 3, 4, and 6 years, respectively.

4. The study period may be set based on the length of a need. For example, if the company needs to purchase some earthmoving equipment to fulfill its obligation on a highway project with an expected duration of four years, the study period could be set to the life of the project.

The logical next question is, "How long of study period should I use?" There are no clear-cut answers to this question. Depending on your assumptions, different methods may lead to different results. Where possible one should try to match actual conditions as closely as possible. Using a study period equal to the least common denominator and repurchasing the alternative is a good solution if one is trying to set a policy as to how often a company is going to replace its computers, such as every two years or every three years. However, it would not be a suitable method to determine which computer a company should purchase for a three-year project. For this type of problem, one should match the study period to the life of the project.

There are three methods of adjusting the lives of the alternatives so they are equal.

Shortening an Alternative's Life

The life of an investment may be shortened by assuming a price at which the alternative may be sold at the end of the shortened life. This assumed sale price is known as the salvage value. For example, a company needs to purchase a computer and the company has narrowed the choices down to two alternatives. The first alternative is to purchase a computer costing $1,000 with a useful life of two years. The second alternative is to purchase a computer costing $1,500 with a useful life of three years. If the study period were two years, one would shorten the life of the more expensive computer to two years by assuming a salvage value for the more expensive computer at the end of the second year. The cash flows for both computers using a two-year study period are shown in Figure 17-1.

Figure 17-1 Cash Flow for Computers with a Two-Year Study Period
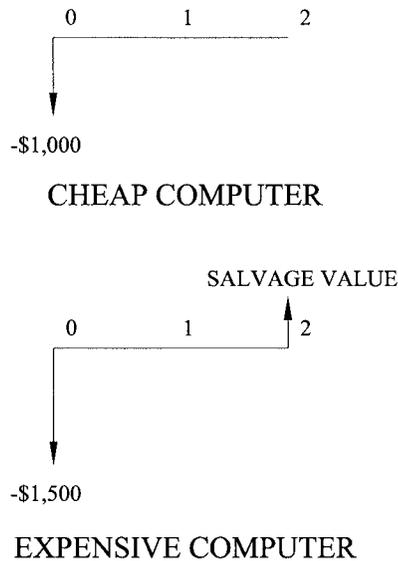
Because these cash flows now cover the same period of time, they may be compared based on equivalence. The weakness of this method is that it assumes a salvage value, which may or may not reflect the value of the asset at the end of the study period.

Lengthening an Alternative's Life

The life of an investment may be lengthened by increasing the maintenance costs of the alternative or investing money into the alternative to upgrade it. For example, the computers in Figure 17-1 could be compared with a three-year study period by lengthening the life of the less expensive computer to three years by adding the cost to upgrade the computer at the end of the second year. The cash flows for both computers using a three-year study period are shown in Figure 17-2.

Because these cash flows now cover the same period of time, they may be compared based on equivalence. The weakness of this method is that it assumes a maintenance or upgrade cost, which may or may not reflect the actual costs.

Repurchasing an Alternative

The life of an investment may be lengthened by repurchasing the alternative until the study period is equaled or exceeded. If the study period is exceeded, a salvage value may be assumed to allow the life of the alternative to equal the study period. For example, the computers in Figure 17-1 could be compared with a six-year study period by repeatedly purchasing the cheaper computer three times over the six-year period and the more expensive computer two times over the same six-year period. The shortest repurchasing option where all of the alternatives end at the same time can be found by determining the least common multiple. The

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        0         1         2         3
        |---------|---------|---------|
        |                   |
        ↓                   ↓
                        UPGRADE COST
    -$1,000
```

CHEAP COMPUTER

```
        0         1         2         3
        |---------|---------|---------|
        |
        |
        ↓
    -$1,500
```

EXPENSIVE COMPUTER
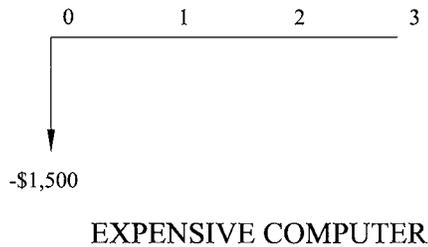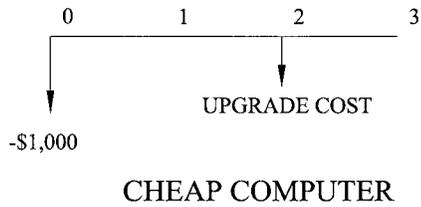
least common multiple is determined by factoring the life of the options into their prime numbers and including the maximum number of each prime number in the least common multiple. For example, if you have an alternative with a life of 8 years and one with a life of 10 years the least common multiple is 40 years. This can be determined as follows. Factoring 8 into its prime numbers, we get $2 \times 2 \times 2$, and factoring 10 into its prime numbers, we get 2×5. The maximum number of twos in either of the lives is 3; therefore, 3 twos are included in the least common multiple. The maximum number of fives in either of the lives is 1; therefore, 1 five is included in the least common multiple. The five combined with the twos give us $2 \times 2 \times 2 \times 5$, which equals 40. The cash flows for both computers using a six-year study period are shown in Figure 17-3.

Because these cash flows now cover the same period of time, they may be compared on equivalence. If the same two computers were evaluated over an

```
        0     1     2     3     4     5     6
        |-----|-----|-----|-----|-----|-----|
        |           |           |
        ↓           ↓           ↓
    -$1,000     -$1,000     -$1,000
```

CHEAP COMPUTER

```
        0     1     2     3     4     5     6
        |-----|-----|-----|-----|-----|-----|
        |                 |
        ↓                 ↓
    -$1,500           -$1,500
```

EXPENSIVE COMPUTER
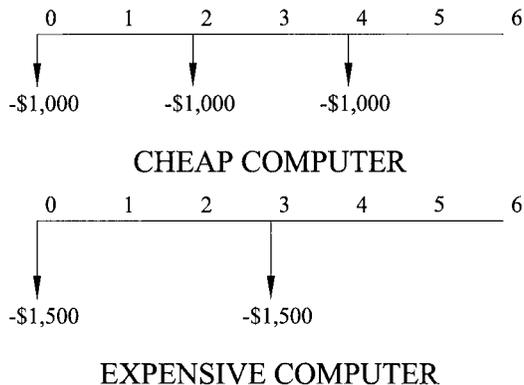
FIGURE 17-4   Cash Flow for Computers with an Eight-Year Study Period

CHEAP COMPUTER

EXPENSIVE COMPUTER
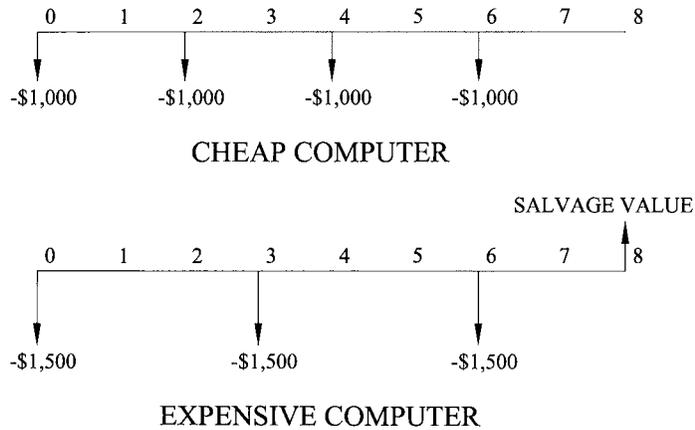
eight-year period, the cheaper computer would be purchased four times during the study period and the expensive computer would be purchased three times during the study period. At the end of the study period the expensive computer would still have one year left in its useful life; therefore, its life would have to be reduced by including the salvage value at the end of the eighth year. The cash flows for both computers using an eight-year study period are shown in Figure 17-4.

Because these cash flows now cover the same period of time, they may be compared based on equivalence. The weakness of this method is that it assumes that we can repeatedly purchase the same item, with the same cash flows in the future. This is often not the case. Investments and products change over time. Let's look at different qualitative methods for analyzing decisions.

NET PRESENT VALUE OR PRESENT WORTH

The net present value (NPV) or present worth method compares alternatives based on their present values at the time of the initial investment at the MARR. The net present value is calculated by determining the equivalent value of the cash receipts and disbursements at the time of the initial investment using the equations in Chapter 15. If the net present value is positive, the alternative produces a return greater than the MARR. If the net present value is zero, the alternative produces a return equal to the MARR. If the net present value is negative, the alternative produces a return less than the MARR and, if possible, the investment should be rejected.

> **Example 17-3:**   Your company is looking at purchasing a front-end loader at a cost of $120,000. The loader would have a useful life of five years with a salvage value of $12,000 at the end of the fifth year. The loader can be billed out at $95.00 per hour. It costs $30.00 per hour to operate the front-end loader and $25.00 per hour for the operator. Using 1,200 billable hours

FIGURE 17-5 Cash Flow for
Example 17-3
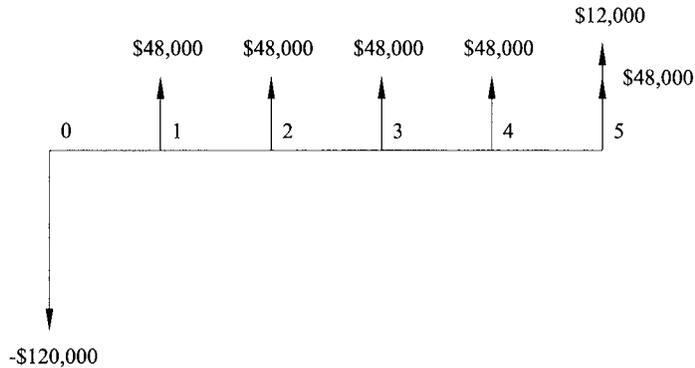
per year determine the net present value for the purchase of the loader using a MARR of 20%. Should your company purchase the loader?

Solution: The hourly profit on the loader equals the billing rate less the operation cost and the cost of the operator. The hourly profit is calculated as follows:

Hourly Profit = $95.00 − $30.00 − $25.00 = $40.00 per hour

The annual profit on the loader equals the hourly profit times the number of billable hours per year and is calculated as follows:

Annual Profit = $40.00/hour(1,200 hour/year) = $48,000/year

The cash flow for the purchase of the loader in Example 17-3 is shown in Figure 17-5.

The present value of the annual profits (P_{AP}) is determined by using Eq. (15-9) as follows:

$$P_{AP} = \$48,000[(1 + 0.20)^5 - 1]/[0.20(1 + 0.20)^5] = \$143,549$$

The present value of the annual profit is positive because it is a cash receipt.

The present value of the salvage value (P_{SV}) is determined by using Eq. (15-3) as follows:

$$P_{SV} = \$12,000/(1 + 0.20)^5 = \$4,823$$

The present value of the salvage value is positive because it is a cash receipt.

The present value purchase price (P_{PP}) of the loader is equal to the purchase price because the net present value is measured at the time of the initial investment. The present value of the purchase price is negative because it is a cash disbursement.

The net present value for purchasing the loader equals the sum of the present values of the individual cash flows and is calculated as follows:

$$NPV = \$143,549 + \$4,823 + (-120,000) = \$28,372$$

Because the net present value is greater than zero, the purchase of the front-end loader will produce a return greater than the MARR and your company should invest in the front-end loader.

When comparing two alternatives with positive net present values, the alternative with the largest net present value produces the most profit in excess of the MARR.

Example 17-4: Your company is looking at purchasing a front-end loader and has narrowed the choice down to two loaders, the loader in Example 17-3 and a smaller, less powerful loader that costs $110,000. The smaller loader would have a useful life of five years with a salvage value of $10,000 at the end of the fifth year. The smaller loader can be billed out at $90.00 per hour. It costs $28.00 per hour to operate the smaller front-end loader and $25.00 per hour for the operator. Using 1,200 billable hours per year determine the net present value for the purchase of the smaller loader using a MARR of 20%. Should your company invest in the smaller loader or the loader in Example 17-3?

Solution: The hourly and annual profits on the smaller loader are calculated in the same manner as they were in Example 17-3 and are as follows:

Hourly Profit = $90.00 − $28.00 − $25.00 = $37.00 per hour
Annual Profit = $37.00/hour(1,200 hour/year) = $44,400/year

The present value of the annual profits and the salvage value are determined by using Equations (15-9) and (15-3) as follows:

$$P_{AP} = \$44,400[(1 + 0.20)^5 - 1]/[0.20(1 + 0.20)^5] = \$132,783$$

$$P_{SV} = \$10,000/(1 + 0.20)^5 = \$4,019$$

The net present value for purchasing the loader is calculated as follows:

$$NPV = \$132,783 + \$4,019 + (-\$110,000) = \$26,802$$

The net present value for purchasing the larger loader in Example 17-3 is $28,372. Because the larger loader has a higher net present value, it is a better financial alternative and your company should purchase the larger loader.

When comparing two investments that require different initial investments, the underlying assumption is that the difference between the initial investments is invested at the MARR. In Example 17-4, the difference in purchase price between the two pieces of equipment was $10,000. Had this money been invested at the MARR of 20% compounded annually, it would have been worth $24,883 at the end of the fifth year. The interest accumulation on the $10,000 is shown in Table 17-4.

SIDEBAR 17-1

CALCULATING THE NET PRESENT VALUE USING EXCEL

Example 17-3 may be set up in a spreadsheet as shown in the following figure:

	A	B	C	D
1	Year	Cashflow		MARR
2	0	-120,000.00		20%
3	1	48,000.00		
4	2	48,000.00		Net Present Value
5	3	48,000.00		$28,371.91
6	4	48,000.00		
7	5	60,000.00		

To set up this spreadsheet, the following formulas, text, and values need to be entered into it:

	A	B	C	D
1	Year	Cashflow		MARR
2	0	-120000		0.2
3	1	48000		
4	2	48000		Net Present Value
5	3	48000		=NPV(D2,B3,B4,B5,B6,B7)+B2
6	4	48000		
7	5	60000		

The spreadsheet uses the NPV function to calculate the net present value of the payments occurring at the end of years 1 through 5. The NPV function is written as

$$=NPV(rate, value1, value2, . . .)$$

where

> rate = minimal attractive rate of return
> value 1 = cash flow at the end of period 1
> value 2 = cash flow at the end of period 2

The NPV function is not set up to handle cash flows occurring at the end of period 0 (the beginning of period 1); therefore, the cash flow that occurs at the end of period 0 must be added to the net present value.

TABLE 17-4   Interest Accumulation on $10,000

YEAR	BEGINNING BALANCE ($)	INTEREST ($)	ENDING BALANCE ($)
1	10,000	2,000	12,000
2	12,000	2,400	14,400
3	14,400	2,880	17,280
4	17,280	3,456	20,736
5	20,736	4,147	24,883

The present value of $24,883 in five years at an interest rate of 20% is calculated using Eq. (15-3) as follows:

$$P = \$24,883/(1 + 0.20)^5 = \$10,000$$

In the analysis of the smaller loader, one could have included an additional $10,000 investment at 20% occurring at the time the loader was purchased, which represents the difference in the purchase price of the two loaders. This investment would generate a cash receipt in the fifth year of $24,883. The present value of this cash receipt is $10,000 and will offset the initial $10,000 investment, leaving the net present value unchanged. Had the difference in purchase prices been invested at a rate other than the MARR, the cash receipt would not have offset the initial investment and the net present value would have changed. Investing the difference in purchase prices in an investment with a return less than the MARR would decrease the net present value, whereas investing the difference in purchase prices in an investment with a return greater than the MARR would increase the net present value.

Some cash flows include only cash disbursements or costs. In this case, the net present value is often referred to as the net present cost. Only cost may be used when the alternatives provide similar services or revenues. For example, when selecting a computer for your office staff, it would be very hard to determine the revenues associated with the purchase of the computer. As long as the computers being compared provide similar benefits they may be compared based on costs. This eliminates the need to estimate revenues and greatly simplifies the calculations. With only costs in the cash flow, the net present cost will always be negative. Here, the objective is to minimize the net present cost or choose the alternative with the least net present cost. This type of analysis is used for a life-cycle cost analysis, where we look not only at the purchase costs of an item (for example, a computer system or building), but also at the costs to own and operate the item throughout its useful life. The following is an example of a life-cycle cost analysis for a heating system.

Example 17-5:   Your company needs to purchase a new furnace and has narrowed the choices down to two furnaces. The first furnace cost $3,000 and has an estimated annual operation cost of $1,050. The second furnace cost $3,500 and has an estimated annual operation cost of $950. The estimated life

FIGURE 17-6 Cash Flows for Example 17-5
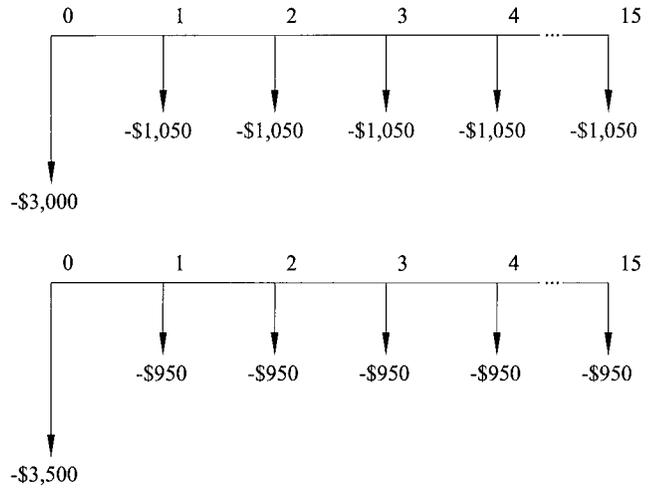
of both furnaces is fifteen years. Using a MARR of 15%, what is the net present cost of each of these furnaces? Which furnace should your company purchase?

Solution: The cash flows for the two furnaces in Example 17-5 are shown in Figure 17-6.

The present value of the annual costs (P_{AC}) for the first furnace is determined by using Eq. (15-9) as follows:

$$P_{AC} = -\$1,050[(1 + 0.15)^{15}-1]/[0.15(1 + 0.15)^{15}]$$
$$P_{AC} = -\$6,140$$

The present value of the purchase price of the first furnace is equal to the purchase price because the net present value is measured at the time of the initial investment. The net present cost of purchasing the first furnace is calculated as follows:

$$NPV = P_{AC} + P_{PP} = -\$6,140 + (-\$3,000) = -\$9,140$$

The present value of the annual costs for the second furnace is determined by using Eq. (15-9) as follows:

$$P_{AC} = -\$950[(1 + 0.15)^{15} - 1]/[0.15(1 + 0.15)^{15}] = -\$5,555$$

The present value purchase price of the second furnace is equal to the purchase price. The net present cost of purchasing the second furnace is calculated as follows:

$$NPV = P_{AC} + P_{PP} = -\$5,555 + (-\$3,500) = -\$9,055$$

Your company should purchase the second furnace because it has the least net present cost.

Selection of the study period can have a great effect on the results of a net present value analysis. Because different study periods require different adjustments in the life spans and these adjustments require different assumptions, it is not uncommon to get conflicting results when selecting between two alternatives while analyzing them with different study periods. A longer study period may favor one of the options, whereas a shorter study period may favor another option.

Let's look at a few examples in which the purchase of a new dump truck is compared to the purchase of a used dump truck. The new dump truck has a useful life of seven years. The used dump truck has a useful life of four years. In the first example the study period will be matched to the useful life of the new dump truck and the used dump truck will be overhauled to extend its life to seven years.

Example 17-6: Your company needs to purchase a dump truck and has narrowed the selection down to two alternatives. The first alternative is to purchase a new dump truck for $65,000. At the end of the seventh year the salvage value of the new dump truck is estimated to be $15,000. The second alternative is to purchase a used dump truck for $50,000. At the end of the fourth year the used dump truck will need an overhaul, which is estimated to cost $20,000. At the end of the seventh year the salvage value of the used dump truck is estimated to be $5,000. The annual profits, revenues less operation costs, are $17,000 per year for either truck. Using a MARR of 18% and a seven-year study period, calculate the net present value for each of the dump trucks. Which truck should your company purchase?

Solution: The present value of the annual profits for either truck is determined by using Eq. (15-9) as follows:

$$P_{AP} = \$17{,}000[(1 + 0.18)^7 - 1]/[0.18(1 + 0.18)^7] = \$64{,}796$$

The present value of the salvage value for the new dump truck is determined by using Eq. (15-3) as follows:

$$P_{SV} = \$15{,}000/(1 + 0.18)^7 = \$4{,}709$$

The net present value for the purchase of the new dump truck is calculated as follows:

$$NPV = \$64{,}796 + \$4{,}709 + (-\$65{,}000) = \$4{,}505$$

The present value of the overhaul (P_O) and the salvage value for the used dump truck are determined by using Eq. (15-3) as follows:

$$P_O = -\$20{,}000/(1 + 0.18)^4 = -\$10{,}316$$

$$P_{SV} = \$5{,}000/(1 + 0.18)^7 = \$1{,}570$$

The net present value for the purchase of the used dump truck is calculated as follows:

$$NPV = \$64{,}796 + (-\$10{,}316) + \$1{,}570 + (-\$50{,}000) = \$6{,}050$$

The used truck has the highest NPV; therefore, your company should purchase the used truck.

In the second example the study period will be matched to the useful life of the used dump truck.

Example 17-7: Your company needs to purchase a dump truck and has narrowed the selection down to two alternatives. The first alternative is to purchase a new dump truck for $65,000. At the end of the fourth year the salvage value of the new dump truck is estimated to be $40,000. The second alternative is to purchase a used dump truck for $50,000. At the end of the fourth year the salvage value of the used dump truck is estimated to be $5,000. The annual profits, revenues less operation costs, are $17,000 per year for either truck. Using a MARR of 18% and a four-year study period, calculate the net present value for each of the dump trucks. Which truck should your company purchase?

Solution: The present value of the annual profits for either truck is determined by using Eq. (15-9) as follows:

$$P_{AP} = \$17{,}000[(1 + 0.18)^4 - 1]/[0.18(1 + 0.18)^4] = \$45{,}731$$

The present value of the salvage value for the new dump truck is determined by using Eq. (15-3) as follows:

$$P_{SV} = \$40{,}000/(1 + 0.18)^4 = \$20{,}632$$

The net present value for the purchase of the new dump truck is calculated as follows:

$$NPV = \$45{,}731 + \$20{,}632 + (-\$65{,}000) = \$1{,}363$$

The present value of the salvage value for the used dump truck is determined by using Eq. (15-3) as follows:

$$P_{SV} = \$5{,}000/(1 + 0.18)^4 = \$2{,}579$$

The net present value for the purchase of the used dump truck is calculated as follows:

$$NPV = \$45{,}731 + \$2{,}579 + (-\$50{,}000) = -\$1{,}690$$

The new truck has the highest NPV; therefore, your company should purchase the new truck.

In Examples 17-6 and 17-7 we see that we selected different alternatives because of the different estimated cash flows used in the net present value analysis.

Let's look at another example. In this example each truck will be repurchased until their useful lives end during the same year, which will occur in the twenty-eighth year.

Example 17-8: Your company needs to purchase a dump truck and has narrowed the selection down to two alternatives. The first alternative is to purchase a new dump truck for $65,000. At the end of the seventh year the salvage value of the new dump truck is estimated to be $15,000. The second alternative is to purchase a used dump truck for $50,000. At the end of the fourth year the salvage value of the used dump truck is estimated to be $5,000. The annual profits, revenues less operation costs, are $17,000 per year for either truck. Using a MARR of 18% and a twenty-eight year study period, calculate the net present value for each of the dump trucks. Which truck should your company purchase?

Solution: The present value of the annual profits for either truck is determined by using Eq. (15-9) as follows:

$$P_{AP} = \$17{,}000[(1 + 0.18)^{28} - 1]/[0.18(1 + 0.18)^{28}] = \$93{,}527$$

The present value of the salvage values for the new dump truck is determined by summing the present value of salvage values occurring in years 7, 14, 21, and 28. The present value for each salvage value is calculated using Eq. (15-3) as follows:

$$P_{SV7} = \$15{,}000/(1 + 0.18)^{7} = \$4{,}709$$
$$P_{SV14} = \$15{,}000/(1 + 0.18)^{14} = \$1{,}478$$
$$P_{SV21} = \$15{,}000/(1 + 0.18)^{21} = \$464$$
$$P_{SV28} = \$15{,}000/(1 + 0.18)^{28} = \$146$$
$$P_{SV} = \$4{,}709 + \$1{,}478 + \$464 + \$146 = \$6{,}797$$

The present value of the purchase prices for the new dump truck is determined by summing the present value of purchase prices occurring in years 0, 7, 14, and 21. The present value for each purchase price is calculated using Eq. (15-3) as follows:

$$P_{PP0} = -\$65{,}000/(1 + 0.18)^{0} = -\$65{,}000$$
$$P_{PP7} = -\$65{,}000/(1 + 0.18)^{7} = -\$20{,}405$$
$$P_{PP14} = -\$65{,}000/(1 + 0.18)^{14} = -\$6{,}406$$
$$P_{PP21} = -\$65{,}000/(1 + 0.18)^{21} = -\$2{,}011$$
$$P_{PP} = -\$65{,}000 + (-\$20{,}405) + (-\$6{,}406) + (-\$2{,}011)$$
$$P_{PP} = -\$93{,}822$$

The net present value for the purchase of the new dump truck is calculated as follows:

$$NPV = \$93{,}527 + \$6{,}797 + (-\$93{,}822) = \$6{,}502$$

The present value of the salvage values for the used dump truck is determined by summing the present value of salvage values occurring in years 4, 8, 12, 16, 20, 24, and 28. The present value for each salvage value is calculated using Eq. (15-3) as follows:

$$P_{SV4} = \$5{,}000/(1 + 0.18)^{4} = \$2{,}579$$
$$P_{SV8} = \$5{,}000/(1 + 0.18)^{8} = \$1{,}330$$

$$P_{SV12} = \$5,000/(1 + 0.18)^{12} = \$686$$
$$P_{SV16} = \$5,000/(1 + 0.18)^{16} = \$354$$
$$P_{SV20} = \$5,000/(1 + 0.18)^{20} = \$183$$
$$P_{SV24} = \$5,000/(1 + 0.18)^{24} = \$94$$
$$P_{SV28} = \$5,000/(1 + 0.18)^{28} = \$49$$
$$P_{SV} = \$2,579 + \$1,330 + \$686 + \$354 + \$183 + \$94 + \$49$$
$$P_{SV} = \$5,275$$

The present value of the purchase prices for the used dump truck is determined by summing the present value of purchase prices occurring in years 0, 4, 8, 12, 16, 20, and 24. The present value for each purchase price is calculated using Eq. (15-3) as follows:

$$P_{PP0} = -\$50,000/(1 + 0.18)^{0} = -\$50,000$$
$$P_{PP4} = -\$50,000/(1 + 0.18)^{4} = -\$25,789$$
$$P_{PP8} = -\$50,000/(1 + 0.18)^{8} = -\$13,302$$
$$P_{PP12} = -\$50,000/(1 + 0.18)^{12} = -\$6,861$$
$$P_{PP16} = -\$50,000/(1 + 0.18)^{16} = -\$3,539$$
$$P_{PP20} = -\$50,000/(1 + 0.18)^{20} = -\$1,825$$
$$P_{PP24} = -\$50,000/(1 + 0.18)^{24} = -\$941$$
$$P_{PP} = -\$50,000 + (-\$25,789) + (-\$13,302) + (-\$6,861)$$
$$+ (-\$3,539) + (-\$1,825) + (-\$941)$$
$$P_{PP} = -\$102,257$$

The net present value for the purchase of the used dump truck is calculated as follows:

$$\text{NPV} = \$93,527 + \$5,275 + (-\$102,257) = -\$3,455$$

The new truck has the highest NPV; therefore, your company should purchase the new truck.

INCREMENTAL NET PRESENT VALUE

In Example 17-4 we choose between two mutually exclusive alternatives with different initial capital outlays, $110,000 and $120,000. The incremental net present value is another way to select between mutually exclusive alternatives with different initial costs. The initial cost is the cash disbursement in year 0. The incremental net present value compares two alternatives at a time. The alternative with the higher initial cost is compared to the alternative with the lower initial cost based on the net present value of the difference in the cash receipts and disbursements for the two alternatives. If the incremental net present value is positive, it is financially attractive to invest the extra money and purchase the more expensive alternative. If the incremental net present value is zero, both investments are equally attractive. If incremental net present value is negative, it is financially attractive to purchase the less expensive piece of equipment.

Performing an incremental net present value analysis involves the following steps. First, rank all of the alternatives in order of initial cost from lowest to highest. Second, identify the alternative with the lowest initial cost as the current best alternative. Third, compare the current best alternative with the alternative with the next higher initial cost based on the incremental net present value of the difference in the cash receipts and disbursements for the two alternatives. If the incremental net present value is zero or negative, the current best alternative remains the current best alternative and the competing alternative is eliminated from consideration. If the incremental net present value is positive, the competing alternative becomes the current best alternative and the former current best alternative is eliminated from consideration. This process is repeated until all alternatives have been compared. The current best alternative at the end of the comparison process becomes the selected alternative.

Example 17-9:   Your company is looking at purchasing a new front-end loader and has narrowed the choice down to four loaders. The purchase price, annual profit, and salvage value at the end of five years for each of the loaders is found in Table 17-5. Which front-end loader should your company purchase based on the incremental net present values using a MARR of 20% and a useful life of five years?

Solution: The first step is to rank the alternative in order of initial cost (purchase price). The loaders are compared in the following order: Loader A, Loader C, Loader B, and Loader D. Because Loader A has the lowest initial cost it is designated the current best alternative.

Next, compare Loader A to Loader C. The difference in the purchase price is $10,000 ($120,000 − $110,000). The difference in annual profit is $3,000 ($40,000 − $37,000). The difference in salvage value is $2,000 ($12,000 − $10,000). The difference in the cash flows for these two alternatives is shown in Figure 17-7.

The present value of the difference in annual profits is determined by using Eq. (15-9) as follows:

$$P_{AP} = \$3,000[(1 + 0.20)^5 - 1]/[0.20(1 + 0.20)^5] = \$8,972$$

The present value of the difference in salvage values is determined by using Eq. (15-3) as follows:

$$P_{SV} = \$2,000/(1 + 0.20)^5 = \$804$$

TABLE 17-5   Cash Flow for Example 17-9

CASH FLOW	LOADER A ($)	LOADER B ($)	LOADER C ($)	LOADER D ($)
Purchase Price	110,000	127,000	120,000	130,000
Annual Profit	37,000	43,000	40,000	44,000
Salvage Value	10,000	13,000	12,000	13,000

FIGURE 17-7 Incremental Cash Flow for Loaders A and C
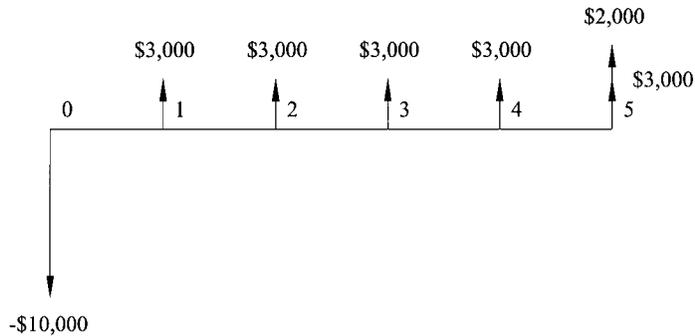
The incremental net present value for the purchase of Loader C in lieu of Loader A is calculated as follows:

$$\text{NPV} = \$8,972 + \$804 + (-\$10,000) = -\$224$$

Because the incremental net present value is negative, Loader A continues to be the current best alternative. Next, we compare Loader A to Loader B, the loader with the next lowest initial cost.

The difference in the purchase price is $17,000 ($127,000 − $110,000). The difference in annual profit is $6,000 ($43,000 − $37,000). And the difference in salvage value is $3,000 ($13,000 − $10,000). The difference in the cash flows for these two alternatives is shown in Figure 17-8.

The present value of the difference in annual profits is determined by using Eq. (15-9) as follows:

$$P_{AP} = \$6,000[(1 + 0.20)^5 - 1]/[0.20(1 + 0.20)^5] = \$17,944$$

The present value of the difference in salvage values is determined by using Eq. (15-3) as follows:

$$P_{SV} = \$3,000/(1 + 0.20)^5 = \$1,206$$

The incremental net present value for the purchase of Loader B in lieu of Loader A is calculated as follows:

$$\text{NPV} = \$17,944 + \$1,206 + (-\$17,000) = \$2,150$$

FIGURE 17-8 Incremental Cash Flow for Loaders A and B
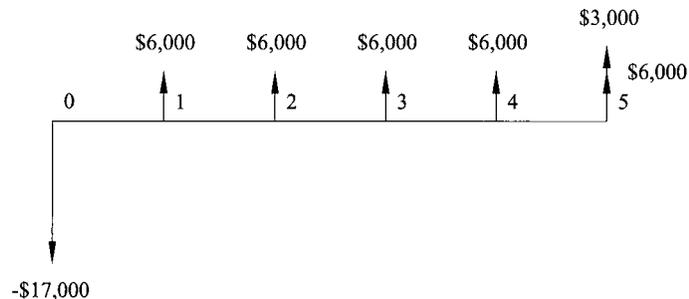
FIGURE 17-9   Incremental
Cash Flow for Loaders B
and D
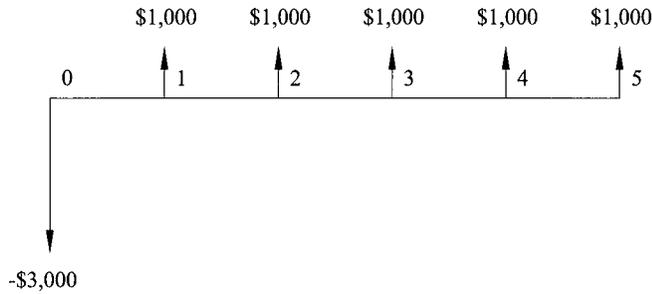
Because the incremental net present value is positive, Loader B becomes the new current best alternative and Loader A is eliminated from comparison. Next, we compare Loader B to Loader D, the only remaining loader not compared.

The difference in the purchase price is $3,000 ($130,000 − $127,000). The difference in annual profit is $1,000 ($44,000 − $43,000). And the difference in salvage value is zero ($13,000 − $13,000). The difference in the cash flows for these two alternatives is shown in Figure 17-9.

The present value of the difference in annual profits is determined by using Eq. (15-9) as follows:

$$P_{AP} = \$1,000[(1 + 0.20)^5 - 1]/[0.20(1 + 0.20)^5] = \$2,991$$

The incremental net present value for the purchase of Loader B in lieu of Loader D is calculated as follows:

$$NPV = \$2,991 + \$0 + (-\$3,000) = -\$8$$

Because the incremental net present value is negative, Loader B continues to be the current best alternative. With no other alternative to compare, Loader B is the selected alternative; therefore, your company should purchase Loader B.

When comparing two alternatives, the incremental net present value equals the difference between the net present values for the alternatives. Let's calculate the net present values for Loader A and Loader C.

The present value of the annual profits for Loader A is determined by using Eq. (15-9) as follows:

$$P_{AP} = \$37,000[(1 + 0.20)^5 - 1]/[0.20(1 + 0.20)^5] = \$110,653$$

The present value of the salvage value for Loader A is determined by using Eq. (15-3) as follows:

$$P_{SV} = \$10,000/(1 + 0.20)^5 = \$4,019$$

The net present value for the purchase of Loader A is calculated as follows:

$$NPV = \$110,653 + \$4,019 + (-\$110,000) = \$4,672$$

The present value of the annual profits for Loader C is determined by using Eq. (15-9) as follows:

$$P_{AP} = \$40{,}000[(1 + 0.20)^5 - 1]/[0.20(1 + 0.20)^5] = \$119{,}624$$

The present value of the salvage values for Loader C is determined by using Eq. (15-3) as follows:

$$P_{SV} = \$12{,}000/(1 + 0.20)^5 = \$4{,}823$$

The net present value for the purchase of Loader C is calculated as follows:

$$NPV = \$119{,}624 + \$4{,}823 + (-\$120{,}000) = \$4{,}447$$

The difference in net present value between Loaders A and C is $-\$225$ ($\$4{,}447 - \$4{,}672$), which is the incremental net present value for Loaders A and C. The small difference is due to rounding errors. Because we change the current best option only when the incremental net present value is positive, the incremental net present value analysis will select the alternative with the highest net present value.

If the "do nothing" alternative is an acceptable alternative, it must be included in the list of alternatives. Failure to include the "do nothing" alternative may result in the selected alternative having a negative net present value. Because the net present value of any of the alternatives is not included in the calculations, one would have no way of knowing if the net present value of the selected alternative is positive. By including the "do nothing" alternative, we are guaranteed to have an alternative with a nonnegative (zero) net present value.

FUTURE WORTH

The future worth method compares alternatives based on their future values at the end of the study period. Future worth is calculated by determining the equivalent value of the cash receipts and disbursements at the end of the study period using the equations in Chapter 15. If the future worth is positive, the alternative produces a return greater than the MARR. If the future worth is zero, the alternative produces a return equal to the MARR. If the future worth is negative, the alternative produces a return less than the MARR and, if possible, the investment should be rejected. Because any present value can be converted to a future value by Equations (15-1) and (15-2) and $(1 + i)^n$ equals a constant for any given i and n, the future worth produces the same result as the net present value for any given i and n.

Example 17-10: Your company is looking at purchasing the front-end loader in Example 17-3 at cost of $120,000. The loader would have a useful life of five years with a salvage value of $12,000 at the end of the fifth year.

The loader can be billed out at $95.00 per hour. It costs $30.00 per hour to operate the front-end loader and $25.00 per hour for the operator. Using 1,200 billable hours per year determine the future worth for the purchase of the loader using a MARR of 20%. Should your company purchase the loader?

Solution: The future value of the purchase price is determined by using Eq. (15-1) as follows:

$$F_{PP} = -\$120,000(1 + 0.20)^5 = -\$298,598$$

The future value of the purchase price is negative because it is a cash disbursement.

The hourly profit on the loader equals the billing rate less the operation cost and the cost of the operator and is calculated as follows:

$$\text{Hourly Profit} = \$95.00 - \$30.00 - \$25.00 = \$40.00 \text{ per hour}$$

The annual profit on the loader equals the hourly profit times the number of billable hours per year and is calculated as follows:

$$\text{Annual Profit} = \$40.00/\text{hour}(1,200 \text{ hours/year}) = \$48,000/\text{year}$$

The future value of the annual profits is determined by using Eq. (15-5) as follows:

$$F_{AP} = \$48,000[(1 + 0.20)^5 - 1]/0.20 = \$357,197$$

The future value of the annual profits is positive because it is a cash receipt.

The future value of the salvage value is equal to the salvage value because the future value is measured at the end of the study period. The future value of the salvage value is positive because it is a cash receipt.

The future worth for purchasing the loader equals the sum of the future values of the individual cash flows and is calculated as follows:

$$\text{FW} = -\$298,598 + \$357,197 + \$12,000 = \$70,599$$

Because the future worth is greater than zero, the purchase of the front-end loader will produce a return greater than the MARR and your company should invest in the front-end loader.

Because all of the costs for Example 17-10 are the same as Example 17-3, the future worth could be calculated from the net present value determined in Example 17-3 using Eq. (15-1) as follows:

$$\text{FW} = \$28,372(1 + 0.20)^5 = \$70,599$$

We see that the net present value and the future worth produce the same result and are related by Eq. (15-1).

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<div style="background:gray">S I D E B A R   1 7 - 2</div>

CALCULATING THE FUTURE WORTH USING EXCEL

Example 17-10 may be set up in a spreadsheet as shown in the following figure:

	A	B	C
1	MARR	20%	
2			
3	Year	Cash Flow	
4	0	-120,000.00	-298,598.40
5	1	48,000.00	99,532.80
6	2	48,000.00	82,944.00
7	3	48,000.00	69,120.00
8	4	48,000.00	57,600.00
9	5	60,000.00	60,000.00
10		Future Worth	70,598.40

To set up this spreadsheet, the following formulas, text, and values need to be entered into it:

	A	B	C
1	MARR	0.2	
2			
3	Year	Cash Flow	
4	0	-120000	=-FV(B1,5-A4,0,B4)
5	1	48000	=-FV(B1,5-A5,0,B5)
6	2	48000	=-FV(B1,5-A6,0,B6)
7	3	48000	=-FV(B1,5-A7,0,B7)
8	4	48000	=-FV(B1,5-A8,0,B8)
9	5	60000	=-FV(B1,5-A9,0,B9)
10		Future Worth	=SUM(C4:C9)

The spreadsheet uses the FV function from Sidebar 15-1 to calculate the future value of each of the cash flows. The future worth is calculated by summing the future value for the cash flows. The FV function assumes that the cash flow used to calculate the future value flows in the direction opposite that of the result of the function; for example, if the cash flow is a disbursement the future value is a receipt. Because we are converting a cash flow to its equivalent at some future time, a negative sign must be placed in front of the FV function to maintain the cash flow in the same direction.

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ANNUAL EQUIVALENT

The annual equivalent method compares alternatives based on their equivalent annual receipts less the equivalent annual disbursements. The annual equivalent is calculated by converting the cash receipts and disbursements into a uniform series of annual cash flows occurring over the study period using the equations in Chapter 15. If the annual equivalent is positive, the alternative produces a return greater than the MARR. If the annual equivalent is zero, the alternative produces a return equal to the MARR. If the annual equivalent is negative, the alternative produces a return less than the MARR and, if possible, the investment should be rejected.

Because any present value can be converted to a uniform series by Equations (15-11) and (15-12) and $i(1 + i)^n/[(1 + i)^n - 1]$ equals a constant for any given i and n, the annual equivalent produces the same result as the net present value for any given i and n. Similarly, because any future value can be converted to a uniform series by Equations (15-7) and (15-8) and $i/[(1 + i)^n - 1]$ equals a constant for any given i and n, the annual equivalent produces the same result as the future value for any given i and n.

The annual equivalent method assumes that they are repurchased until the useful lives end in the same period. When dealing with alternatives of different lives, one of the big advantages of using the annual equivalent method over net present value or future value methods is that it produces the same result using a single life for each of the alternatives as assuming each of the alternatives are repurchases until all of their lives end in the same period. Performing the calculation necessary to complete an annual equivalent analysis using the life of the alternatives is much simpler than performing a net preset value on multiple repurchases of the alternatives. In Example 17-11 we compare the same two alternatives as were compared in Example 17-8. In Example 17-8 they are compared based on net present value using a study period of twenty-eight years. In Example 17-11 they are compared based on annual equivalent. The study period is different for both trucks and is matched to their useful life.

> **Example 17-11:**   Your company needs to purchase a dump truck and has narrowed the selection down to two alternatives. The first alternative is to purchase a new dump truck for $65,000. At the end of the seventh year the salvage value of the new dump truck is estimated to be $15,000. The second alternative is to purchase a used dump truck for $50,000. At the end of the fourth year the salvage value of the used dump truck is estimated to be $5,000. The annual profits, revenues less operation costs, are $17,000 per year for either truck. Using a MARR of 18% calculate the annual worth for each of the dump trucks. Which truck should your company purchase?

Solution: The useful life of the new truck is seven years, which is used as the study period for the new truck. The purchase price for the new truck is converted to a uniform series of annual cash flows by Eq. (15-11) as follows:

$$A_{PP} = -\$65{,}000[0.18(1 + 0.18)^7]/[(1 + 0.18)^7 - 1] = -\$17{,}054$$

The salvage value for the new truck is converted to a uniform series of annual cash flows by Eq. (15-7) as follows:

$$A_{SV} = \$15{,}000(0.18)/[(1 + 0.18)^7 - 1] = \$1{,}235$$

The annual profits for the new truck are already a uniform series. The annual equivalent (AE) for purchasing the loader equals the sum of the uniform series representing each of the individual cash flows and is calculated as follows:

$$AE = -\$17{,}054 + \$1{,}235 + \$17{,}000 = \$1{,}181$$

The useful life of the used truck is four years, which is used as the study period for the used trucks. The purchase price for the used truck is converted to a uniform series of annual cash flows by Eq. (15-11) as follows:

$$A_{PP} = -\$50{,}000[0.18(1 + 0.18)^4]/[(1 + 0.18)^4 - 1] = -\$18{,}587$$

The salvage value for the used truck is converted to a uniform series of annual cash flows by Eq. (15-7) as follows:

$$A_{SV} = \$5{,}000(0.18)/[(1 + 0.18)^4 - 1] = \$959$$

The annual profits for the used truck are already a uniform series. The annual equivalent for purchasing the truck equals the sum of the uniform series representing each of the individual cash flows and is calculated as follows:

$$AE = -\$18{,}587 + \$959 + \$17{,}000 = -\$628$$

The new truck has the highest annual equivalent; therefore, your company should purchase the new truck.

If we convert the net present value for the new truck from Example 17-8 to a uniform series of annual cash flows over the combined life of twenty-eight years by Eq. (15-11) we get the following:

$$A = \$6{,}502[0.18(1 + 0.18)^{28}]/[(1 + 0.18)^{28} - 1] = \$1{,}182$$

This value is the same as the annual equivalent calculated for the new truck; however, the calculations are much simpler using the annual equivalent. The same is true for the used truck.

CALCULATING THE ANNUAL EQUIVALENT USING EXCEL

Example 17-11 may be set up in a spreadsheet as shown in the following figure:

	A	B
1	MARR:	18%
2	Purchase Price:	-65,000.00
3	Salvage Value:	15,000.00
4	Life:	7
5	Annual Profit (Cost):	17,000.00
6	Annual Equivalent:	1,181.90

To set up this spreadsheet, the following formulas, text, and values need to be entered into it:

	A	B
1	MARR:	0.18
2	Purchase Price:	-65000
3	Salvage Value:	15000
4	Life:	7
5	Annual Profit (Cost):	17000
6	Annual Equivalent:	=-PMT(B1,B4,B2,B3)+B5

The spreadsheet uses the PMT function from Sidebar 15-4 to calculate the annual equivalent for the purchase price and salvage value. The PMT function assumes that the cash flows used to calculate the annual equivalent (payment) flow in the direction opposite that of the result of the function. Because we are converting a cash flow at the beginning and end of a number of periods to an annual equivalent series, a negative sign must be placed in front of the PMT function to maintain the cash flow in the correct direction.

RATE OF RETURN

The rate of return is also known as the internal rate of return or return on investment. The rate of return is the interest rate paid on the unrecovered balance of an investment over the remaining life of the investment. The rate of return is the interest rate that produces a net present value of zero. The rate of return is calculated in the same manner that the unknown interest rate was calculated in Examples 15-13, 15-14, and 15-15. When using the rate of return to choose

between investment alternatives, the alternative with the largest rate of return is selected.

If the rate of return on an alternative is greater than the MARR the net present value, future worth, and annual equivalent are greater than zero for the alternative. If the rate of return on an alternative is equal to the MARR the net present value, future worth, and annual equivalent are all equal to zero for the alternative. If the rate of return on an alternative is less than the MARR the net present value, future worth, and annual equivalent are less than zero for the alternative.

Example 17-12: Your company is looking at purchasing a front-end loader at a cost of $120,000. The loader would have a useful life of five years with a salvage value of $12,000 at the end of the fifth year. The loader can be billed out at $95.00 per hour. It costs $30.00 per hour to operate the front-end loader and $25.00 per hour for the operator. Using 1,200 billable hours per year determine the rate of return for the purchase of the loader. If your company's MARR was 20%, should your company purchase the loader?

Solution: The hourly profit on the loader equals the billing rate less the operation cost and the cost of the operator and is calculated as follows:

Hourly Profit = $95.00 − $30.00 − $25.00 = $40.00 per hour

The annual profit on the loader equals the hourly profit times the number of billable hours per year and is calculated as follows:

Annual Profit = $40.00/hour(1,200 hours/year) = $48,000/year

The cash flow for the purchase of the loader in Example 17-12 is shown in Figure 17-10.

The present value of the annual profits is determined by using Eq. (15-9) as follows:

$$P_{AP} = \$48,000[(1 + i)^5 - 1]/[i\,(1 + i)^5]$$

The present value of the annual profits is positive because it is a cash receipt.
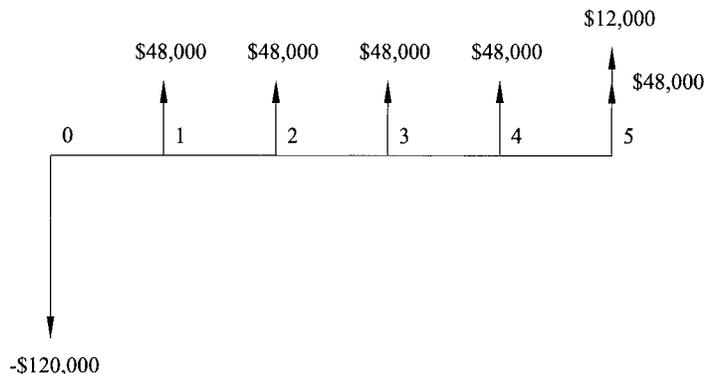
S I D E B A R   1 7 - 4

CALCULATING THE RATE OF RETURN USING EXCEL

Example 17-12 may be set up in a spreadsheet as shown in the following figure:

	A	B	C	D
1	Year	Cash Flow		Rate of Return
2	0	-120,000.00		30.06%
3	1	48,000.00		
4	2	48,000.00		
5	3	48,000.00		
6	4	48,000.00		
7	5	60,000.00		

To set up this spreadsheet, the following formulas, text, and values need to be entered into it:

	A	B	C	D
1	Year	Cash Flow		Rate of Return
2	0	-120000		=IRR(B2:B7,0.2)
3	1	48000		
4	2	48000		
5	3	48000		
6	4	48000		
7	5	60000		

The spreadsheet uses the IRR function to calculate the rate of return. The IRR function is written as

=IRR(values,guess)

where

values = values used to calculate the rate of return; with the first value representing the cash flow at the end of period 0, the second representing the cash flow at the end of period 1, and so forth
guess = your guess at the rate of return

There must be both positive and negative cash flows in the series for the IRR function to work. When cash flows switch directions more than once during the series, the IRR function may have multiple solutions. Once the IRR function finds a solution, it stops looking for additional solutions. The value guessed will determine where the IRR function begins looking for a solution and, as such, will determine which of the possible solution it finds.

The present value of the salvage value is determined by using Eq. (15-3) as follows:

$$P_{SV} = \$12,000/(1 + i)^5$$

The present value of the salvage value is positive because it is a cash receipt.

The present value of the purchase price of the loader is equal to the purchase price because the net present value is measured at the time of the initial investment. The present value of the purchase price is negative because it is a cash disbursement.

To find the rate of return one must write an equation that includes the present values representing all of the cash flows and set the net present value to zero. The net present value for purchasing the loader equals the sum of the present values of the individual cash flows and is calculated as follows:

$$0 = \$48,000[(1 + i)^5 - 1]/[i(1 + i)^5] + \$12,000/(1 + i)^5 + (-\$120,000)$$

Solving for i by trial and error we find that i equals 30.06%. The rate of return for the investment is 30.06%. Because the rate of return is greater than the MARR of 20%, the present value will be positive and your company should invest in the front-end loader.

One weakness of the rate of return method is that a series of cash flows may have multiple rates of returns if one of the following conditions exist: the cash flow in year zero is positive, the sign on the net cash flow changes from period to period more than once, or the disbursements are greater than the receipts.

When using the rate of return method to select between alternatives one must be sure to convert them to mutually exclusive alternatives as shown in the following example.

Example 17-13: Your company has $200,000 to invest and has identified the following three investments. Investment A requires an initial investment of $120,000 and has an annual rate of return of 18%. Investment B requires an initial investment of $80,000 and has an annual rate of return of 22%. Investment C requires an initial investment of $20,000 and has an annual rate of return of 35%. Unused funds will be placed in a bank account with an annual percentage yield of 6%. You may invest in each of the investments only once. All of the investments have a one year life. Which investments should your company invest in?

Solution: In this example one must first combine the investments into mutually exclusive alternatives. A list of the mutually exclusive alternatives is shown in Table 17-6. The option to not invest in any of the investments, the "do nothing" alternative, is included as Alternative 1. For each of these alternatives the unused capital will be placed in a bank account, which earns 6% interest; therefore, whether to invest the excess funds does not need to be included in the alternatives.

TABLE 17-6 Mutually Exclusive Alternatives

ALTERNATIVE	INVESTMENTS	ACCEPTABLE
1	None	Yes
2	A	Yes
3	B	Yes
4	C	Yes
5	A & B	Yes
6	B & C	Yes
7	A & C	Yes
8	A, B, & C	No

Only Alternative 8 is not acceptable because of the limited funds. Alternative 8 would require the investment of $220,000 and the company only has $200,000 available.

The interest earned on Investments A, B, and C is determined by using Eq. (16-1) as follows:

$$I_A = \$120,000(0.18)1 = \$21,600$$
$$I_B = \$80,000(0.22)1 = \$17,600$$
$$I_C = \$20,000(0.35)1 = \$7,000$$

For Alternative 1 the rate of return is 6% because all of the money is invested in a bank account that has an annual percentage yield of 6%.

For Alternative 2, $120,000 will be invested in Investment A and $80,000 in a bank account that has an annual percentage yield of 6%. The interest earned on the bank account is calculated using Eq. (16-1) as follows:

$$I_{Bank} = \$80,000(0.06)1 = \$4,800$$

The interest earned for Alternative 2 is $26,400 ($21,600 + $4,800), for a total worth of $226,400 at the end of year 1. Using Eq. (15-1) to set the cash investment equivalent to the value of the investment at the end of year 1 we get the following:

$$\$226,400 = \$200,000(1 + i)^1$$

Solving for i we find that i equals 13.2%.

Alternatives 3 through 7 are solved in a similar manner—adding up the interest on the investments and the bank account and then determining the rate of return. Alternative 8 is not a feasible alternative because we lack the funds to invest in all three investments. Table 17-7 shows the rate of return for each of the alternatives. Alternative 5, investment in Investments A and B, has the highest rate of return and is the alternative that should be selected.

In Example 17-13, had we begun by investing in the investment with the highest rate of return and continued doing so until we ran out of money, we would have invested in Investments A and C, which would have a combined

TABLE 17-7   Rate of Return for Alternatives for Example 17-13

ALTERNATIVE	INVESTMENTS	ROR (%)
1	None	6.0
2	A	13.2
3	B	12.4
4	C	8.9
5	A & B	19.6
6	B & C	15.3
7	A & C	16.1
8	A, B, & C	NA

return of 16.1%. From Example 17-13 we see the set of investments with the highest return is the combination of Investments A and B, with a combined return of 19.6%. This underscores the importance of setting up mutually exclusive alternatives to be analyzed.

INCREMENTAL RATE OF RETURN

Like the incremental net present value, the incremental rate of return is a way to select between mutually exclusive alternatives with different initial costs. The incremental rate of return compares alternatives by determining the rate of return on the difference in initial costs. The initial cost is the cash disbursement in year 0. The incremental rate of return is determined by setting the incremental net present value to zero. The incremental rate of return compares two alternatives at a time. The alternative with the higher initial cost is compared with the alternative with the lower cost based on the rate of return for the difference in the cash receipts and disbursements for the two alternatives. If the incremental rate of return is greater than the MARR, it is financially attractive to invest the more expensive alternative. If the incremental rate of return is equal to the MARR, both alternatives are equally attractive. If the incremental rate of return is less than the MARR, it is financially attractive to invest the less expensive alternative.

Performing an incremental rate of return analysis involves the following steps. First, rank all of the alternatives in order of initial cost. Second, identify the alternative with the lowest initial cost as the current best alternative. Third, compare the current best alternative with the alternative with the next higher initial cost based on its incremental rate of return. If the incremental rate of return is equal to or less than the MARR, the current best alternative remains the current best alternative and the competing alternative is eliminated from consideration. If the incremental rate of return is greater than the MARR, the competing alternative becomes the new current best alternative and the former current best alternative is eliminated from consideration. This process is repeated until all alternatives

have been compared. The current best alternative at the end of the comparison process becomes the selected alternative. Let's look at Example 17-9 using the incremental rate of return method in lieu of the incremental net present value.

Example 17-14: Your company is looking at purchasing a new front-end loader and has narrowed the choice down to four loaders. The useful life of the loaders is five years. The purchase price, annual profit, and salvage value at the end of five years for each of the loaders is found in Table 17-8. Which front-end loader should your company purchase based on the incremental rate of return and a MARR of 20%?

Solution: The first step is to rank the alternatives in order of initial capital outlay. The loaders are compared in the following order: Loader A, Loader C, Loader B, and Loader D. Because Loader A has the lowest purchase price it is designated the current best alternative.

Next, compare Loader A to Loader C. The difference in the purchase price is $10,000 ($120,000 − $110,000). The difference in annual profit is $3,000 ($40,000 − $37,000). The difference in salvage value is $2,000 ($12,000 − $10,000).

The present value of the difference in annual profits is determined by using Eq. (15-9) as follows:

$$P_{AP} = \$3,000[(1 + i)^5 - 1]/[i(1 + i)^5]$$

The present value of the difference in salvage values is determined by using Eq. (15-3) as follows:

$$P_{SV} = \$2,000/(1 + i)^5$$

The incremental net present value for the purchase of the new loader is calculated as follows:

$$\text{NPV} = \$3,000[(1 + i)^5 - 1]/[i(1 + i)^5] + \$2,000/(1 + i)^5 + (-\$10,000)$$

Setting the incremental net present value to zero and solving by trial and error we find the incremental rate of return equals 19.05%. Because the incremental rate of return is less than the MARR, Loader A continues to be the current best alternative.

Next, we compare Loader A to Loader B, the loader with the next lowest cost. The difference in the purchase price is $17,000 ($127,000 − $110,000).

TABLE 17-8   Cash Flow for Example 17-14

CASH FLOW	LOADER A ($)	LOADER B ($)	LOADER C ($)	LOADER D ($)
Purchase Price	110,000	127,000	120,000	130,000
Annual Profit	37,000	43,000	40,000	44,000
Salvage Value	10,000	13,000	12,000	13,000

The difference in annual profit is $6,000 ($43,000 − $37,000). The difference in salvage value is $3,000 ($13,000 − $10,000).

The present value of the difference in annual profits is determined by using Eq. (15-9) as follows:

$$P_{AP} = \$6,000[(1 + i)^5 - 1]/[i(1 + i)^5]$$

The present value of the difference in salvage values is determined by using Eq. (15-3) as follows:

$$P_{SV} = F/(1 + i)^n = \$3,000/(1 + i)^5$$

The incremental net present value for the purchase of the new loader is calculated as follows:

$$NPV = \$6,000[(1 + i)^5 - 1]/[i(1 + i)^5] + \$3,000/(1 + i)^5$$
$$+ (-\$17,000)$$

Setting the incremental net present value to zero and solving by trial and error we find the incremental rate of return equals 25.32%. Because the incremental rate of return is greater than the MARR, Loader B is the new current best alternative and Loader A is eliminated from comparison.

Next, we compare Loader B to Loader D, the only remaining loader not compared. The difference in the purchase price is $3,000 ($130,000 − $127,000). The difference in annual profit is $1,000 ($44,000 − $43,000). The difference in salvage value is zero ($13,000 − $13,000).

The present value of the difference in annual profits is determined by using Eq. (15-9) as follows:

$$P_{AP} = \$1,000[(1 + i)^5 - 1]/[i(1 + i)^5]$$

The incremental net present value for the purchase of the new loader is calculated as follows:

$$NPV = \$1,000[(1 + i)^5 - 1]/[i(1 + i)^5] + \$0 + (-\$3,000)$$

Setting the incremental net present value to zero and solving by trial and error we find the incremental rate of return equals 19.86%. Because the incremental rate of return is less than the MARR, Loader B continues to be the current best alternative. With no other alternative to compare, Loader B is the selected alternative; therefore, your company should purchase Loader B.

The incremental rate of return produced the same results as the incremental net present value and the net present value. Because we change the current best option only when the incremental rate of return is greater than the MARR, which corresponds to an incremental positive net present value, the incremental rate of return selects the alternative with the highest net present value.

If the "do nothing" alternative is an acceptable alternative it must be included in the list of alternatives. Failure to include the "do nothing" alternative may result in the selected alternative having a rate of return less than the MARR,

which corresponds to a negative net present value. Because the rate of return of any of the alternatives is not included in the calculations, one would have no way of knowing if the net present value of the selected alternative is positive. By including the "do nothing" alternative, we are guaranteed to have an alternative with a nonnegative net present value.

CAPITAL RECOVERY WITH RETURN

The capital recovery with return method is similar to the annual equivalent method, except it looks only at capital costs. The capital costs included in the calculations are the purchase price and salvage value. When using the capital recovery with return method the purchase price and salvage value are converted to their annual equivalents at the MARR. Because the purchase price is almost always larger than the salvage value, the capital recovery with return is usually negative. The capital recovery with return represents the annual equivalent loss in the value of an asset. For an alternative to be financially attractive the loss in value of the asset must be offset by profits (revenue less noncapital expenses) on the alternative. The capital recovery with return is useful when looking at the effects of pricing and volume (e.g., billing hours) on the attractiveness of an alternative.

Example 17-15: Your company is looking at purchasing a new hydraulic excavator. The excavator has a purchase price of $120,000, a useful life of five years, and a salvage value of $12,000 at the end of the fifth year. The excavator can be billed out at $95.00 per hour. It costs $30.00 per hour to operate the excavator and $25.00 per hour for the operator. Using a MARR of 20% and the capital recovery with return method, determine the minimum number of billable hours in a year that will make purchasing the excavator financially attractive. How many hours will need to be billed if the billing rate were reduced to $90.00 per hour?

Solution: For purchase of the excavator to be financially attractive, the annual profit must be equal to the capital recovery with return, which is equal to the annual equivalents of the purchase price and salvage value at the MARR.

The purchase price for the excavator is converted to a uniform series of annual cash flows by Eq. (15-11) as follows:

$$A_{PP} = -\$120,000[0.20(1 + 0.20)^5]/[(1 + 0.20)^5 - 1]$$
$$A_{PP} = -\$40,126$$

The salvage value for the excavator is converted to a uniform series of annual cash flows by Eq. (15-7) as follows:

$$A_{SV} = \$12,000(0.20)/[(1 + 0.20)^5 - 1] = \$1,613$$

The capital recovery with return (CR) for purchasing the excavator equals the sum of the uniform series representing each of the individual cash flows and is calculated as follows:

$$CR = -\$40,126 + \$1,613 = -\$38,513$$

For the purchase of the excavator to be financially attractive, it must generate $38,513 in profit (revenue less noncapital costs) each year. The hourly profit on the excavator equals the billing rate less the operation cost and the cost of the operator and is calculated as follows:

$$\text{Hourly Profit} = \$95.00 - \$30.00 - \$25.00 = \$40.00 \text{ per hour}$$

The annual profit on the loader equals the hourly profit times the number of billable hours per year. Solving for number of billable hours we get the following:

$$\text{Billable Hours} = \text{Annual Profit}/\text{Hourly Profit}$$

Calculating the minimum number of billable hours per year required to offset the capital recovery with return we get the following:

$$\text{Billable Hours} = \$38,513 \text{ per year}/\$40.00 \text{ per hour} = 963 \text{ hours/year}$$

At a billing rate of $95.00 per hour, one would need to bill 963 hours per year. If the billing rate were reduced to $90.00 per hour, the hourly profit on the excavator would be:

$$\text{Hourly Profit} = \$90.00 - \$30.00 - \$25.00 = \$35.00 \text{ per hour}$$

Calculating the minimum number of billable hours per year required to offset the capital recovery with return we get the following:

$$\text{Billable Hours} = \$38,513 \text{ per year}/\$35.00 \text{ per hour}$$
$$\text{Billable Hours} = 1,100 \text{ hours/year}$$

At a billing rate of $90.00 per hour, one would need to bill 1,100 hours per year or an additional 137 $(1,100 - 963)$ hours per year.

In Example 17-15, if one believes that the excavator can generate a profit of $38,513 per year, then the investment is financially attractive. We showed that this could be done by billing out 963 hours per year at $95.00 per hour or 1,100 hours per year at $90.00 per hour. If we do not think that the excavator can generate a profit of $38,513 per year, then the investment is not financially attractive.

PAYBACK PERIOD WITHOUT INTEREST

The payback period without interest compares alternatives based on how long it takes to pay back the initial costs. The payback period is determined by finding the first period when the sum of the net cash flows, both receipts and disbursements, to date is nonnegative (zero or positive). The payback period without interest ignores the interest on the initial costs. Unless the payback period equals or exceeds

the life of the alternative it also ignores the salvage value. If the payback period is less than or equal to the useful life of the alternative, the alternative will recover the initial costs. Because the time value of money is not included in the calculations, this does not guarantee that the alternative will produce a specific return.

Example 17-16: Your company is looking at purchasing a front-end loader at a cost of $120,000. The loader can be billed out at $95.00 per hour. It costs $30.00 per hour to operate the front-end loader and $25.00 per hour for the operator. The useful life of the equipment is five years. Using 1,200 billable hours per year determine the payback period without interest for the front-end loader. Does the front-end loader generate enough revenue to recover the initial cost?

Solution: The hourly profit on the loader equals the billing rate less the operation cost and the cost of the operator and is calculated as follows:

$$\text{Hourly Profit} = \$95.00 - \$30.00 - \$25.00 = \$40.00 \text{ per hour}$$

The annual profit on the loader equals the hourly profit times the number of billable hours per year and is calculated as follows:

$$\text{Annual Profit} = \$40.00/\text{hour}(1,200 \text{ hours/year}) = \$48,000/\text{year}$$

The payback period (n') equals the number of years it takes to recover the initial capital outlay and is calculated as follows:

$$n' = \text{Purchase Price/Annual Profit}$$
$$n' = \$120,000/\$48,000 \text{ per year} = 2.5 \text{ years}$$

The payback period for the investment is three years because the initial cost is paid back during the third year. Because the payback period is less than the five-year useful life of the loader, the purchase of the loader will generate enough revenue to recover the initial costs, the operating costs, and the cost of the operator.

When comparing investments using the payback period without interest method, the alternative with the smallest payback period is selected. The payback period without interest is a good measure of how long the initial investment is at risk and represents how long the initial investment is tied up in an alternative. The weakness of this method is that it ignores interest on the investment, it ignores all cash flows that occur after the payback period, and it ignores the life of the equipment. Because the payback without interest ignores what happens to the investment after the payback period occurs it favors short-term investments. It is a good analytical tool when there is a high degree of uncertainty and one wants to recoup the investment as soon as possible. The payback period without interest is a good supplemental measure to be used in conjunction with other quantitative methods that include analysis of the cash flow over the life of the alternatives, such as the net present value. The payback period without interest can be used to help select between two alternatives with similar net present values by selecting the one that places the initial capital outlay at risk for the shortest period of time.

TABLE 17-9   Investments for Example 17-17

YEAR	INVESTMENT A ($)	INVESTMENT B ($)	INVESTMENT C ($)
0	−1,000	−2,000	−2,000
1	500	500	900
2	500	600	900
3	0	400	900
4	0	500	900
5	0	3,500	900

Example 17-17:   Your company is looking at investing in one of the three investments shown in Table 17-9. Using the payback period without interest method which investment would you choose? How would your decision be different if you used the net present value method and a MARR of 20%? Which investment would you choose if the investments were considered high risks and you used both of the above methods?

Solution: The payback period for Investment A is two years because in year 2 the sum of the cash flows to date (−$1,000 + $500 + $500 = $0) is nonnegative for the first time. The payback period for Investment B is four years because in year 4 the sum of the cash flows to date (−$2,000 + $500 + $600 + $400 + $500 = $0) is nonnegative for the first time. The payback period for Investment C is three years because in year 3 the sum of the cash flows to date (−$2,000 + $900 + $900 + $900 = $,700) is nonnegative for the first time. Based on the payback period without interest Investment A is the most financially attractive investment.

Next, we look at the net present value for each of the investments.

The present value of the uniform series of payments in Investment A is determined by using Eq. (15-9) as follows:

$$P_A = \$500[(1 + 0.20)^2 - 1]/[0.20(1 + 0.20)^2] = \$764$$

The present value of the initial investment is equal to the initial investment. The net present value for Investment A equals the sum of the present values and is calculated as follows:

$$\text{NPV} = \$764 + (-\$1,000) = -\$236$$

The present value of individual cash flows for Investment B is determined by using Eq. (15-3) as follows:

$$P_0 = -\$2,000/(1 + 0.20)^0 = -\$2,000$$
$$P_1 = \$500/(1 + 0.20)^1 = \$417$$
$$P_2 = \$600/(1 + 0.20)^2 = \$417$$
$$P_3 = \$400/(1 + 0.20)^3 = \$231$$
$$P_4 = \$500/(1 + 0.20)^4 = \$241$$
$$P_5 = \$3,500/(1 + 0.20)^5 = \$1,407$$

The net present value for Investment B equals the sum of the present values and is calculated as follows:

$$\text{NPV} = -\$2{,}000 + \$417 + \$417 + \$231 + \$241 + \$1{,}407 = \$713$$

The present value of the uniform series of payments in Investment C is determined by using Eq. (15-9) as follows:

$$P_A = \$900[(1 + 0.20)^5 - 1]/[0.20(1 + 0.20)^5] = \$2{,}692$$

The present value of the initial investment is equal to the initial investment. The net present value for Investment C equals the sum of the present values and is calculated as follows:

$$\text{NPV} = \$2{,}692 + (-\$2{,}000) = \$692$$

Based on the net present value at a MARR of 20%, Investment B is the most financially attractive alternative.

For the third part of this example we look at both the net present value and the payback period without interest. Looking at Investment A we see that it has the shortest payback period but has a negative net present value. This is because the cash flows stop in the same year that the payback period occurs. Because the payback period with interest ignores the cash flows that occur after the payback period and ignores interest on the cash flows, it allows one to accept an investment that one would reject based on its net present value.

Looking at Investments B and C we see that Investment B is preferred based on the net present values of the investments by $21 ($713 − $692); however, Investment C is preferred based on the payback period without interest because its payback period is a year less.

Given that the investments are risky and we would like to get our investment back as soon as possible, we will forgo the additional $21 gained on Investment B and invest in Investment C because our investment is exposed to loss for a shorter period of time.

In Example 17-17 we saw that the payback period without interest could be used in conjunction with the net present value to add a measure of how long an investment is at risk to a measure of the return on the investment.

PAYBACK PERIOD WITH INTEREST

The payback period with interest is the same as the payback period without interest, except it takes the time value of money into account when making the calculations. The payback period with interest compares alternatives based on how long it takes to pay back the initial costs including interest on the initial costs at the MARR. The payback period is determined by finding the first period when the sum of the net cash flows—including interest on the outstanding investment—is

nonnegative (zero or positive). This is done by determining at what point in time the present value of the net cash flows (both cash receipts and disbursements) are equal to or greater than zero. Unless the payback period is equal to or longer than the useful life of the alternative it ignores the salvage value. If the payback period is less than or equal to the useful life of the alternative, the alternative will recover the initial investment plus interest on the investment at the MARR. Because the time value of money is included in the calculation, a payback period that is less than or equal to the useful life of the alternative guarantees that the investment will have a return equal to or greater than the MARR.

Example 17-18: Your company is looking at purchasing a front-end loader at a cost of $120,000. The loader can be billed out at $95.00 per hour. It costs $30.00 per hour to operate the front-end loader and $25.00 per hour for the operator. The useful life of the equipment is five years. Using 1,200 billable hours per year and a MARR of 20% determine the payback period with interest for the front-end loader. Does the front-end loader generate enough revenue to recover the initial cost while providing a return of at least 20%?

Solution: The hourly profit on the loader equals the billing rate less the operation cost and the cost of the operator and is calculated as follows:

$$\text{Hourly Profit} = \$95.00 - \$30.00 - \$25.00 = \$40.00 \text{ per hour}$$

The annual profit on the loader equals the hourly profit times the number of billable hours per year and is calculated as follows:

$$\text{Annual Profit} = \$40.00/\text{hour}(1,200 \text{ hours/year}) = \$48,000/\text{year}$$

Convert the annual profits to their present values. The present value of the annual profits is determined by using Eq. (15-3) as follows:

$$P_0 = -\$120,000/(1 + 0.20)^0 = -\$120,000$$
$$P_1 = \$48,000/(1 + 0.20)^1 = \$40,000$$
$$P_2 = \$48,000/(1 + 0.20)^2 = \$33,333$$
$$P_3 = \$48,000/(1 + 0.20)^3 = \$27,778$$
$$P_4 = \$48,000/(1 + 0.20)^4 = \$23,148$$
$$P_5 = \$48,000/(1 + 0.20)^5 = \$19,290$$

The payback period with interest for purchase of the loader is four years because in the fourth year the present value of the net cash flows to date ($-\$120,000 + \$40,000 + \$33,333 + \$27,778 + \$23,148 = \$4,259$) is positive for the first time. Because the payback period is less than the five-year useful life of the loader, the purchase of the loader will generate enough return to recover the initial costs plus interest on the initial investment at an interest rate greater than 20%. Because interest was included, the payback period of the loader has increased from the three years in Example 17-16 to four years.

When comparing investments using the payback period with interest, the alternative with the smallest payback period is selected. The payback period with interest is a good measure of how long the initial investment plus interest on the investment is at risk and represents how long it takes to recover the initial investment with interest. The weakness of this method is that it ignores the cash flows that occur after the payback period and the life of the equipment. Because the payback period with interest ignores what happens to the investment after the payback period is completed it favors short-term investments. It is a better tool than the payback period without interest because it incorporates interest on the investment. It is a good analytical tool when there is a high degree of uncertainty and one wants to recoup the investment as soon as possible. The payback period with interest is a good supplemental measure to be used in conjunction with other quantitative methods that include analysis of the cash flow over the life of the alternatives, such as the net present value. The payback period with interest can be used to help select between two alternatives with similar net present values by selecting the one that places the initial capital outlay at risk for the shortest period of time.

PROJECT BALANCE

The project balance method is a graphical method that shows the potential profit or exposure to loss for any period during an alternative's life. The project balance incorporates interest at the MARR into its calculations. From the project balance graph, one can read the future worth of the alternative at the end of its life as well as the payback period with interest. The project balance is calculated by determining the net cash flow to date for any alternative including interest on the outstanding investment or project balance (PB) as shown in Example 17-19.

Example 17-19: Your company is looking at purchasing a front-end loader at a cost of $120,000. The loader can be billed out at $95.00 per hour. It costs $30.00 per hour to operate the front-end loader and $25.00 per hour for the operator. The useful life of the equipment is five years. The salvage value of the loader at the end of the fifth year is $12,000. Using 1,200 billable hours per year and a MARR of 20% prepare a project balance graph for the front-end loader. What is the future worth of the loader? What is the payback period with interest?

Solution: The hourly profit on the loader equals the billing rate less the operation cost and the cost of the operator and is calculated as follows:

Hourly Profit = $95.00 − $30.00 − $25.00 = $40.00 per hour

The annual profit on the loader equals the hourly profit times the number of billable hours per year and is calculated as follows:

Annual Profit = $40.00/hour(1,200 hours/year) = $48,000/year

The project balance at the initial point in time ($t = 0$) is a negative $120,000 or the cost of the front-end loader. If we were to terminate the investment immediately after the purchase of the loader, we would be exposed to a $120,000 loss.

The project balance at the end of the first year is equal to the project balance at the end of the previous period plus interest on the project balance from the previous period plus the annual profit. Because the project balance at the end of year 0 was negative the interest for period one will be negative. The project balance at the end of the first year is calculated as follows:

$$PB_1 = PB_0 + PB_0(i) + \text{Annual Profits}$$
$$PB_1 = -\$120,000 + (-\$120,000)(0.20) + \$48,000 = -\$96,000$$

The project balance at the end of the second year is equal to the project balance at the end of the previous period plus interest on the project balance from the previous period plus the annual profit. The project balance at the end of the second year is calculated as follows:

$$PB_2 = -\$96,000 + (-\$96,000)(0.20) + \$48,000 = -\$67,200$$

Similarly, the project balances at the end of the third and fourth years is calculated as follows:

$$PB_3 = -\$67,200 + (-\$67,200)(0.20) + \$48,000 = -\$32,640$$
$$PB_4 = -\$32,640 + (-\$32,640)(0.20) + \$48,000 = \$8,832$$

In the fifth year the annual profit is increased by salvage value of the equipment or $12,000 for an annual profit of $60,000. The project balance at the end of the fifth year is calculated as follows:

$$PB_5 = \$8,832 + \$8,832(0.20) + \$60,000 = \$70,598$$

The annual project balance for the loader may be graphed as shown in Figure 17-11.

From the project balance graph we see the payback period with interest is four years, which is the same as was calculated for this same loader in Example 17-18. We also see that the loader has a future worth of $70,598, which is the same as was calculated in Example 17-10. The one-dollar difference is due to rounding.

The weakness of the project balance method is that it ignores the salvage value until the end of the study period. In Example 17-19, at the initial point in time our graph shows that we would incur a $120,000 loss if we terminated the investment in the loader. This ignores the possibility that we may sell the loader to offset part of this $120,000 loss.

The strength of the project balance method is that it incorporates both the future worth—which is mathematically related to the net present value— and payback period with interest as well as showing the potential profit or potential loss for each of the years. The project balance method is useful for risky
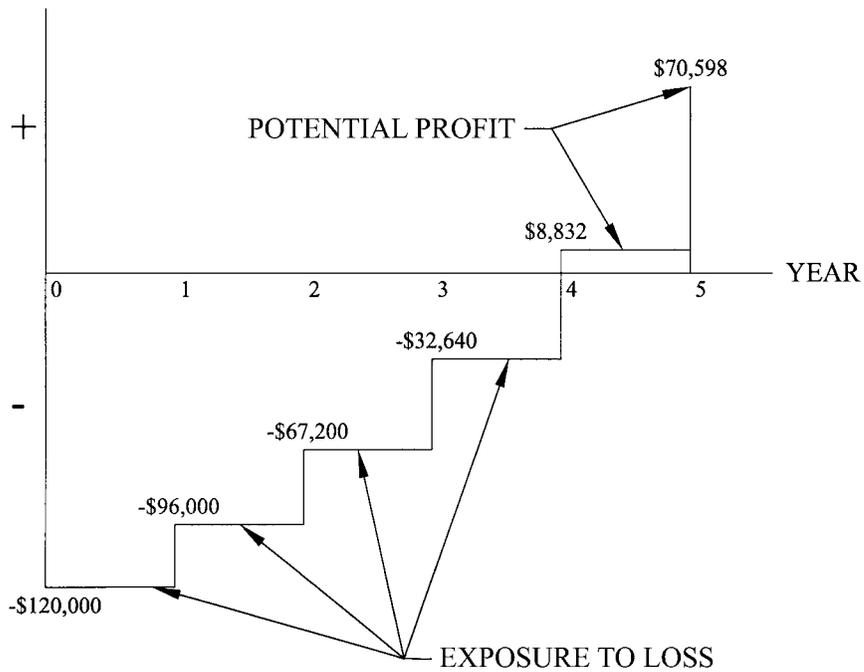
FIGURE 17-11 Project Balance for Example 17-19

investment because it shows the exposure to loss for the project. The project balance method is also useful for comparing projects with similar future worth (or net present value or annual equivalents) and similar payback periods with interest.

Example 17-20: Your company is looking at investing in one of two investments, Investment A and Investment B. Both investments are considered risky and your company wants to recoup its investment as soon as possible. The cash flows for both investments are shown in Table 17-10. Using a MARR of 20% the future worth for Investment A is $63,715 and the

TABLE 17-10 Cash Flows for Example 17-20

YEAR	INVESTMENT A ($)	INVESTMENT B ($)
0	−100,000	−100,000
1	42,000	19,000
2	42,000	48,000
3	42,000	60,000
4	42,000	69,000
5	42,000	21,000

future worth for Investment B is $63,710. Both investments have a payback period with interest of four years at the MARR. Using the project balance method, which investment should your company invest in?

Solution: The project balance for Investment A is calculated as follows:

$PB_0 = -\$100,000$
$PB_1 = -\$100,000 + (-\$100,000)(0.20) + \$42,000 = -\$78,000$
$PB_2 = -\$78,000 + (-\$78,000)(0.20) + \$42,000 = -\$51,600$
$PB_3 = -\$51,600 + (-\$51,600)(0.20) + \$42,000 = -\$19,920$
$PB_4 = -\$19,920 + (-\$19,920)(0.20) + \$42,000 = \$18,096$
$PB_5 = \$18,096 + \$18,096(0.20) + \$42,000 = \$63,715$

The project balance for Investment B is calculated as follows:

$PB_0 = -\$100,000$
$PB_1 = -\$100,000 + (-\$100,000)(0.20) + \$19,000 = -\$101,000$
$PB_2 = -\$101,000 + (-\$101,000)(0.20) + \$48,000 = -\$73,200$
$PB_3 = -\$73,200 + (-\$73,200)(0.20) + \$60,000 = -\$27,840$
$PB_4 = -\$27,840 + (-\$27,840)(0.20) + \$69,000 = \$35,592$
$PB_5 = \$35,592 + \$35,592 (0.20) + \$21,000 = \$63,710$

The project balance graphs for Investments A and B are shown in Figure 17-12.

Based on the project balance graphs we should invest in Investment A because it has a smaller exposure to loss during the first three years.

INVESTMENT A   INVESTMENT B
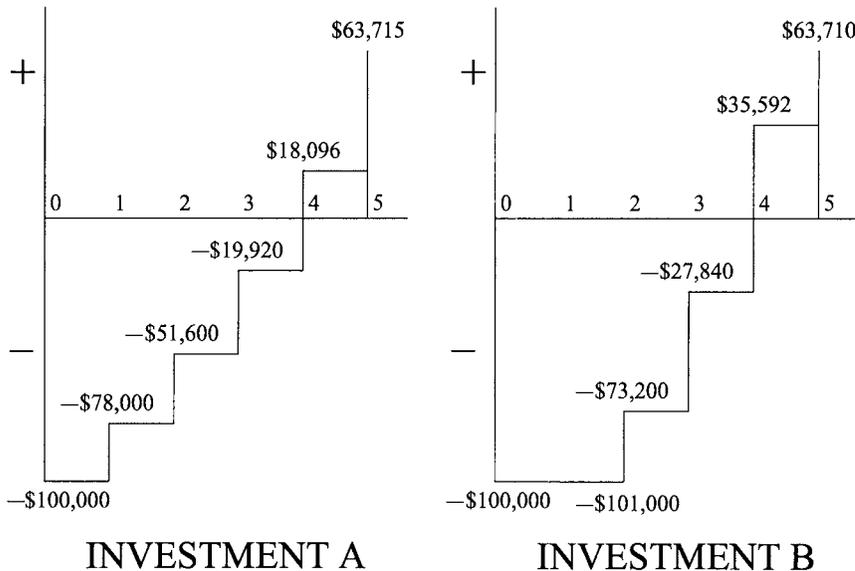
FIGURE 17-12 Project Balance for Example 17-20

NONECONOMIC FACTORS IN DECISION MAKING

Throughout this chapter we looked at the economic factors that make an alternative financially advantageous or financially unviable. When considering alternatives there are other considerations that may sway the decision in one direction or another. These include environmental factors, ergonomic factors, and personal preferences.

CONCLUSION

There are a number of analytical tools that may be used to compare mutually exclusive alternatives. Mutually exclusive alternatives are alternatives where the acceptance of one of the alternatives precludes investing in the other alternatives. To compare alternatives a minimum attractive rate of return (MARR) must be established. Alternatives are worth the effort and risk when they have a return equal to or greater than the MARR. To compare alternatives with different lives, the lives must be made equal. This may be done by shortening the lives of alternatives, lengthening the lives of alternatives, or repurchasing alternatives.

The net present value, incremental net present value, future worth, and annual equivalent methods are related by the equivalence equations and produce the same decisions. When these methods produce a nonnegative numbers the alternative has a return equal to or greater than the MARR. The rate of return for an investment equals the interest rate that produces a net present value equal to zero and may be directly compared to the MARR. The capital recovery method is useful in determining the annual break-even volumes for an alternative. The payback period without interest and payback period with interest select between alternatives based on the speed at which the original investment is recouped and are useful when used in conjunction with the net present value, future worth, annual equivalent, or rate of return. The project balance method incorporates the future value and the payback period with interest while showing the exposure to loss and potential profits graphically for each period.

PROBLEMS

1. A manager has up to $190,000 available to invest in new construction equipment for the company. The manager must purchase a new dump truck and does not have a need for a second dump truck. The dumping trailer can only be purchased along with a dump truck. From the following list of possible equipment, identify all of the mutually exclusive alternatives and identify which of the alternatives are not acceptable.

No.	Description	Cost ($)
1	Loader	125,000
2	Dump Truck 1	70,000
3	Dump Truck 2	65,000
4	Dumping Trailer for the Dump Truck	25,000

2. A manager has up to $200,000 available to invest in new construction equipment for the company. The manager must purchase at least one new dump truck. There are two dumping trailers available. A dump truck must be purchased for each dumping trailer purchased. From the following list of possible equipment, identify all of the mutually exclusive alternatives and identify which of the alternatives are not acceptable.

No.	Description	Cost ($)
1	Dump Truck 1	70,000
2	Dump Truck 2	65,000
3	Dumping Trailer for the Dump Truck	28,000
4	Dumping Trailer for the Dump Truck	25,000

3. Determine the MARR for a company that can borrow funds at 9% and requires 6% profit margin.

4. Determine the MARR for a company that can invest excess funds at 6% and requires 7% profit margin.

5. Your company is looking at purchasing a dump truck at a cost of $65,000. The truck would have a useful life of five years. At the end of the fifth year the salvage value is estimated to be $10,000. The dump truck could be billed out at $55.00 per hour and costs $13.00 per hour to operate. The operator costs $22.00 per hour. Using 1,000 billable hours per year determine the net present value for the purchase of the dump truck using a MARR of 18%. Should your company purchase the dump truck?

6. Your company is looking at purchasing a loader at a cost of $125,000. The loader would have a useful life of seven years. At the end of the seventh year the salvage value is estimated to be $10,000. The loader could be billed out at $85.00 per hour and costs $30.00 per hour to operate. The operator costs $25.00 per hour. Using 1,100 billable hours per year determine the net present value for the purchase of the loader using a MARR of 22%. Should your company purchase the loader?

7. Your company needs to purchase a new track hoe and has narrowed the selection to two pieces of equipment. The first track hoe costs $100,000 and costs $32.00 per hour to operate. The second track hoe costs $110,000 and costs $27.00 per hour to operate. The operator costs $28.00 per hour. The revenue from either track hoe is $95.00 per hour. Using a useful life of four

years, a salvage value equal to 20% of the purchase price, 1,200 billable hours per year, and a MARR of 20%, calculate the NPV for both track hoes. Which track hoe should your company choose?

8. Your company needs to purchase a new track hoe and has narrowed the selection to two pieces of equipment. The first track hoe costs $100,000 and has an hourly operation cost of $31.00 and a $35,000 salvage value at the end of three years. The second track hoe costs $65,000 and has an hourly operation cost of $36.00 and no salvage value at the end of three years. The operator cost is $29.00 per hour. The revenue from either track hoe is $95.00 per hour. Using 1,200 billable hours per year and a MARR of 20%, calculate the NPV for both track hoes. Which track hoe should your company choose?

9. Your company needs to purchase a track hoe and has narrowed the selection to two pieces of equipment. The first track hoe costs $100,000 and has an hourly operation cost of $31.00 and a useful life of four years. At the end of four years its salvage value is $20,000. The second track hoe costs $65,000 and has an hourly operation cost of $36.00 and has a useful life of three years. At the end of three years its salvage value is $10,000. The operator cost is $29.00 per hour. The revenue from either track hoe is $95.00 per hour. Using 1,200 billable hours per year and a MARR of 20%, calculate the net present value for both track hoes. Assume that each option is repurchased until their useful lives end in the same year. Which track hoe should your company choose?

10. Your company needs to purchase a truck and has narrowed the selection to two pieces of equipment. The first truck costs $70,000 and has an hourly operation cost of $13.00 and a useful life of six years. At the end of six years its salvage value is $10,000. The second truck costs $40,000 and has an hourly operation cost of $18.00 and has a useful life of four years. At the end of four years its salvage value is $5,000. The operator cost is $22.00 per hour. The revenue from either truck is $55.00 per hour. Using 1,500 billable hours per year and a MARR of 18%, calculate the net present value for both trucks. Assume that each option is repurchased until their useful lives end in the same year. Which truck should your company choose?

11. Determine the incremental net present value for Problem 7. Which track hoe should your company choose?

12. Determine the incremental net present value for Problem 8. Which track hoe should your company choose?

13. Determine the future worth for Problem 5. Should your company purchase the dump truck?

14. Determine the future worth for Problem 6. Should your company purchase the loader?

15. Determine the annual equivalent for Problem 5. Should your company purchase the dump truck?

16. Determine the annual equivalent for Problem 6. Should your company purchase the loader?

17. Determine the rate of return for Problem 5. Should your company purchase the dump truck?

18. Determine the rate of return for Problem 6. Should your company purchase the loader?

19. Your company has $100,000 to invest and has identified the following three investments. Investment A requires an initial investment of $70,000 and has an annual rate of return of 15%. Investment B requires an initial investment of $80,000 and has an annual rate of return of 21%. Investment C requires an initial investment of $30,000 and has an annual rate of return of 29%. Unused funds will be placed in a bank account with an annual percentage rate of 5%. You may invest in each of the investments only once. All of the investments have a life of one year. Which investment should your company invest in?

20. Your company has $200,000 to invest and has identified the following three investments. Investment A requires an initial investment of $130,000 and has an annual rate of return of 12%. Investment B requires an initial investment of $70,000 and has an annual rate of return of 16%. Investment C requires an initial investment of $30,000 and has an annual rate of return of 27%. Unused funds will be placed in a bank account with an annual percentage rate of 4.5%. You may invest in each of the investments only once. All of the investments have a life of one year. Which investment should your company invest in?

21. Determine the incremental rate of return for Problem 7. Which track hoe should your company choose?

22. Determine the incremental rate of return for Problem 8. Which track hoe should your company choose?

23. Your company has purchased a new track hoe for $100,000. The track hoe can be billed out at $95.00 per hour, has an hourly operation cost of $33.00, and has a useful life of four years. At the end of four years the track hoe has a salvage value of $20,000. The operator cost is $27.00 per hour. Using a MARR of 21%, what is the minimum number of billable hours each year in order for your company to break even?

24. Your company has purchased a new excavator for $210,000. The track hoe can be billed out at $180.00 per hour, has an hourly operation cost of $104.00, and has a useful life of six years. At the end of six years the track hoe has a salvage value of $35,000. The operator cost is $36.00 per hour. Using a MARR of 25%, what is the minimum number of billable hours each year in order for your company to break even?

25. Determine the payback period without interest for Problem 5. If the maximum allowable payback period without interest for your company is three years, should your company purchase the dump truck?

26. Determine the payback period without interest for Problem 6. If the maximum allowable payback period without interest for your company is four years, should your company purchase the loader?

27. Determine the payback period with interest for Problem 5. If the maximum allowable payback period with interest for your company is four years, should your company purchase the dump truck?

28. Determine the payback period with interest for Problem 6. If the maximum allowable payback period with interest for your company is five years, should your company purchase the loader?

29. Draw a project balance chart for Problem 5.

30. Draw a project balance chart for Problem 6.

31. Modify the spreadsheet in Sidebar 17-1 to compare the purchase of two pieces of equipment based on their net present value. Test your spreadsheet by entering the data from Problems 8 and 9. Compare your spreadsheet's solution to the answers to these problems.

32. Modify the spreadsheet in Sidebar 17-4 to compare the purchase of two pieces of equipment based on their rate of return. Test your spreadsheet by entering the data from Problems 8 and 9. Compare your spreadsheet's solution to the answers to these problems.